2.5 Apply the Remainder and Factor Theorems Cont.
2.6 Find Rational Zeros

The Factor Theorem

A polynomial $g(x)$ is a factor of $f(x)$ if...

1. $\dfrac{f(x)}{g(x)}$ has a remainder of $\underline{\ 0\ }$.

2. k is a zero of $g(x)$ and $f(k) = \underline{\ 0\ }$.

ex: Is $g(x)$ a factor of $f(x)$?

a) $g(x) = x - 5, \quad f(x) = x^3 - 7x^2 + 7x + 15$

$$\frac{f(x)}{g(x)}$$

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 7 & 15 \\ & & 5 & -10 & -15 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

Yes!

ex: Is $g(x)$ a factor of $f(x)$?

b) $g(x) = x + 7, \qquad f(x) = x^2 - 9$

$$\begin{array}{r|rrr} -7 & 1 & 0 & -9 \\ & & -7 & 49 \\ \hline & 1 & -7 & 40 \end{array}$$

No

ex: Factor $f(x)$ completely given one of its factors.

a) $f(x) = 15x^3 + x^2 - 22x - 8; \qquad x+1$

$$\begin{array}{r|rrrr} -1 & 15 & 1 & -22 & -8 \\ & & -15 & 14 & 8 \\ \hline & 15 & -14 & -8 & 0 \end{array}$$

$15x^2 - 14x - 8$

$f(x) = \underline{(x+1)(5x+2)(3x-4)}$

ex: Factor $f(x)$ completely given one of its factors.

b) $f(x) = x^3 - 7x^2 + 7x + 15; \qquad x - 3$

$$
\begin{array}{r|rrrr}
3 & 1 & -7 & 7 & 15 \\
 & & 3 & -12 & -15 \\
\hline
 & 1 & -4 & -5 & 0
\end{array}
$$

$x^2 - 4x - 5$

$f(x) = \underline{(x-5)(x+1)(x-3)}$

ex: Find the zeros of $f(x)$ given one of its zeros.

$$f(x) = x^3 + 6x^2 + 9x + 4; \qquad \underline{-4}$$

$$\begin{array}{r|rrrr} -4 & 1 & 6 & 9 & 4 \\ & & -4 & -8 & -4 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$x = -4, \ -1$ mult. of 2

The Rational Zero Theorem

If $f(x)$ is a polynomial then every rational zero of $f(x)$ comes in the form of...

$$\frac{\text{factors of constant}}{\text{factors of L.C.}}$$

leading coefficient

$$\frac{p}{q} \quad y = qx^n + \ldots + p$$

ex: List the possible rational zeros.

a) $f(x) = 2x^3 - 7x^2 + 9$

$$\frac{\text{factors of } 9}{\text{factors of } 2} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 2}$$

$$\pm 1 \quad \pm \frac{1}{2} \quad \pm 3 \quad \pm \frac{3}{2} \quad \pm 9 \quad \pm \frac{9}{2}$$

ex: List the possible rational zeros.

b) $f(x) = 4x^4 - x^3 - 7x^2 + 4x - 2$

$$\frac{\text{factors of } -2}{\text{factors of } 4} = \frac{\pm 1, \pm 2}{\pm 1, \pm 2, \pm 4}$$

$$\boxed{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4} \quad \pm 2}$$

Finding ALL Zeros

Before finding all zeros consider whether the polynomial factors

- if YES, then factor

- if NO, then Rat. Zeo Thm $\left(\frac{p}{q}\right)$

*If the polynomial is QUADRATIC,
then decide on a method

ex: Determine the best method for finding the zeros of the given polynomial

a) $f(x) = 10x^3 - 17x^2 - 7x + 2$

Rat. Zero thm RZT

b) $f(x) = 16x^4 - 54x$

factor

ex: Find all zeros.

a) $f(x) = x^3 + 7x^2 + 15x + 9$

RZT

$\frac{\text{factors } 9}{\text{factors } 1}$

$\pm 1, \pm 3, \pm 9$

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 15 & 9 \\ & & -1 & -6 & -9 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$1x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$x = -3$

$\boxed{-1, -3 \text{ (mult } 2)}$

ex: Find all zeros.

b) $f(x) = x^3 - 9x^2 + 21x - 4$

$$\frac{\text{factors of } 4}{\text{factors of } 1}$$

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1}$$

$$\begin{array}{r|rrrr} 4 & 1 & -9 & 21 & -4 \\ & & 4 & -20 & 4 \\ \hline & 1 & -5 & 1 & 0 \end{array}$$

$x^2 - 5x + 1 = 0$

$x = \frac{5 \pm \sqrt{21}}{2}$

$$\boxed{4, \frac{5 \pm \sqrt{21}}{2}}$$

ex: Find all zeros.

c) $f(x) = x^3 - 3x^2 + 4x - 12$

$= x^2(x-3) + 4(x-3)$

$0 = (x^2+4)(x-3)$

$\downarrow \qquad \downarrow$

$\pm 2i \qquad 3$

$\boxed{3, \pm 2i}$

ex: Find all zeros. $\boxed{-1, 5, -2 \pm i}$

d) $f(x) = x^4 - 16x^2 - 40x - 25$

Poss. rat. zeros:

$\pm 1, \pm 5, \pm 25$

-1	1	0	-16	-40	-25
		-1	1	15	25
	1	-1	-15	-25	0

$x^3 - x^2 - 15x - 25$

5	1	-1	-15	-25
		5	20	25
	1	4	5	0

$x^2 + 4x + 5 = 0$

$(x^2 + 4x + 4) - 4 + 5 = 0$

$\sqrt{(x+2)^2} = \sqrt{-1}$

$|x+2| = i$

$x = -2 \pm i$

Complex Conjugate & Irrational Conjugate Theorem

Imaginary and irrational roots always come in conjugate pairs.

$\sqrt{7}$ $\quad$ $4+i$ $\quad$ $3-\sqrt{6}$

$-\sqrt{7}$ $\quad$ $4-i$ $\quad$ $3+\sqrt{6}$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

a) $-\frac{2}{5}$, $3i$, $-3i$

$(5x+2)(x-3i)(x+3i)$

$(5x+2)(x^2+9) = f(x)$

$5x^3+2x^2+45x+18 = f(x)$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) 0 multiplicity 5, $1-\sqrt{3}$, $1+\sqrt{3}$

$x^5(x^2-2x-2)=f(x)$

Sum: $1-\sqrt{3}+1+\sqrt{3}=2$

Product: $(1-\sqrt{3})(1+\sqrt{3})=-2$

$1-3$

$f(x)=x^7-2x^6-2x^5$

$$y = ax^2 + bx + c$$

$$y = x^2 + 5x + 6$$

$$y = (x + 2)\ (x + 3)$$

Roots: -2 | -3

Formula

Sum of roots = $-2 + -3 = -5 \ \} \ \frac{-b}{a} = \frac{-5}{1}$

Product of roots = $-2 \cdot -3 = 6 \ \} \ \frac{c}{a} = \frac{6}{1}$

Review

ex: Find the error.

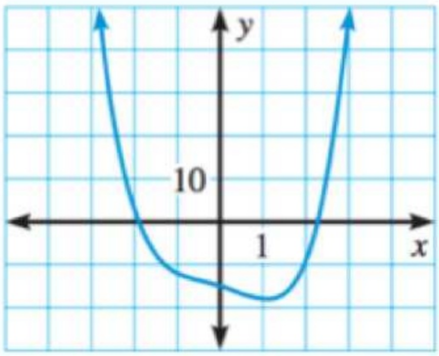
$$\begin{array}{r|rrr} 2 & 1 & -5 & 3 \\ & & 2 & -6 \\ \hline & 1 & -3 & -3 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$

Review

ex: Determine the number of imaginary zeros with the given degree and graph.

Degree: 4



Review

ex: Simplify.

$$\frac{\left(3x^{-2}x^{10}y^{2}\right)^{-1}}{\left(2x^{3}y^{2}\right)^{3}}$$

Review

ex: If $f(3)=0$, which statement(s) about f(x) are true?

a) $x+3$ is a factor of f(x)

b) $x-3$ is a factor of f(x)

c) -3 is a root of f(x)

d) $\dfrac{f(x)}{x-3}$ has a remainder of zero.

Review

ex: Perform the indicated operation.

$$\left(x^3+7x+15\right)-\left(3x^3-6x^2-9x+4\right)$$