(13) $g(x) = x^3 - 31x - 30$

q p

$x = 6, -5, -1$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

1 | 1 0 −31 −30
1 1 −30
1 1 −30

−1 | 1 0 −31 −30
−1 1 30
1 −1 −30 0

$x^2 - x - 30 = 0$

$(x-6)(x+5) = 0$

(47) $V = x(x-1)(x-2)$

$24 = (x^2 - x)(x-2)$

$24 = x^3 - 2x^2 - x^2 + 2x$

$f(x) = x^3 - 3x^2 + 2x - 24$

$\frac{P}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

23.) $f(x) = \underset{q}{2}x^4 - 5x^3 + 10x^2 - \underset{p}{9}$

A. -9
B. $-\frac{1}{2}$
C. $\frac{5}{2}$
D. 3

$-9 : \pm 1, \pm 3, \pm 9$

$2 : \pm 1, \pm 2$

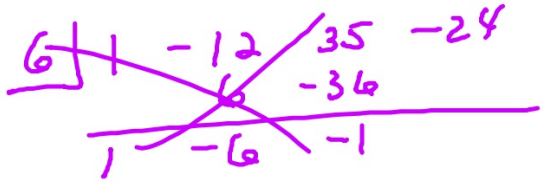
$\frac{p}{q} : \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

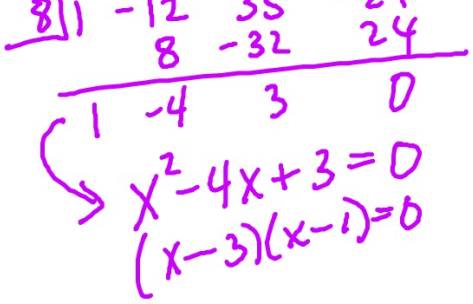
11.) $f(x) = x^3 - 12x^2 + 35x - 24$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$x = 8, 3, 1$

(24)





$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

25) $g(x) = 2x^3 - 7x^2 + 9$

$\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$

$$\begin{array}{r|cccc} -1 & 2 & -7 & 0 & 9 \\ & & -2 & 9 & -9 \\ \hline & 2 & -9 & 9 & 0 \end{array}$$

$2x^2 - 9x + 9 = 0$

$(2x - 3)(x - 3) = 0$

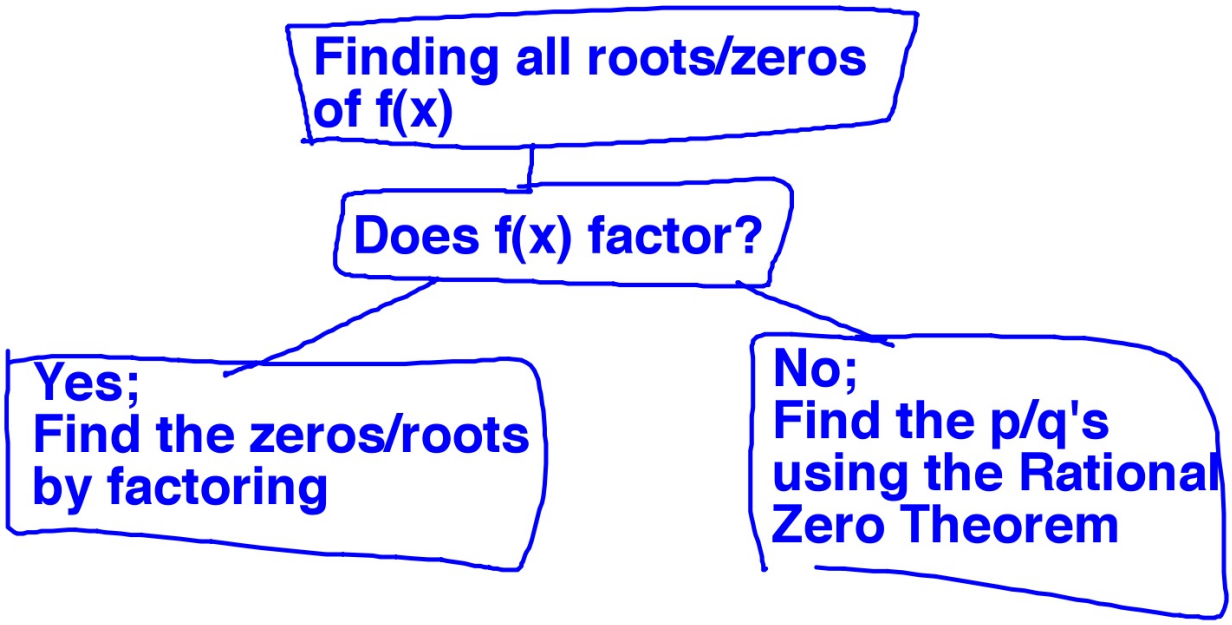
$3/2, \quad 3$

(1) $2x^3 - 54$

$2(x^3 - 27)$

$2(x-3)(x^2+3x+9)$

2.6: Rational Zero Theorem (continued)



① $f(x) = x^3 - 3x^2 + 4x - 12$ **Find the zeros.**

$0 = x^3 - 3x^2 + 4x - 12$

$0 = x^2(x-3) + 4(x-3)$

$0 = (x^2+4)(x-3)$

$x^2+4=0$ $\quad x=3$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$3, \pm 2i$

(2) $f(x) = 2x^3 - 3x^2 - 14x + 15$ **Find the zeros.**

$\frac{-5}{2}$ | 2 −3 −14 15

−5 20 −15

2 −8 6 0

$\frac{-5}{2}$, 3, 1

$2x^2 - 8x + 6 = 0$

$2(x^2 - 4x + 3) = 0$

$2(x-3)(x-1) = 0$

Find the zeros.

(3) $f(x) = x^4 - x^3 - 8x^2 + 6x + 12$

2	1	−1	−8	6	12
		2	2	−12	−12
	1	1	−6	−6	0

$x^3 + x^2 - 6x - 6 = 0$

$x^2(x+1) - 6(x+1) = 0$

$(x^2 - 6)(x+1) = 0$

$2, -1, \pm\sqrt{6}$

Write a polynomial function (in standard form) of least degree with integral coefficients with the given roots.

$\frac{1}{2}, \sqrt{3}, -\sqrt{3}$

$(2x-1)\underbrace{(x-\sqrt{3})(x+\sqrt{3})}$

$(2x-1)(x^2-3)$

$f(x)=2x^3-x^2-6x+3$

conjugate pairs for irrational numbers and imaginary numbers

Complex conjugate theorem:
If a + bi is a root, so is a - bi.

Complex roots come in conjugate pairs

Irrational Conjugates Theorem:
Irrational roots come in conjugate pairs

$2+\sqrt{3}$, $2-\sqrt{3}$

Write a polynomial function (in standard form) of least degree with integral coefficients.

4, -7 Sum: -3 product: -28

$x^2 + 3x - 28 = f(x)$

$x^2 - bx + c$

$-b = -\text{sum}$

$c = \text{product}$

-4, -5 sum: -9 product: 20

$f(x) = x^2 + 9x + 20$

Write a polynomial function of least degree (in standard form) with integral coefficients.

$1 \pm \sqrt{3}$

$1+\sqrt{3}, \; 1-\sqrt{3}$

Sum: 2

product: -2

$(1+\sqrt{3})(1-\sqrt{3})$

$1-3$

-2

$$f(x) = x^2 - 2x - 2$$

$$\frac{2 \pm \sqrt{4-4(1)(-2)}}{2(1)}$$

$$\frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3} \; \text{U}$$

Write a polynomial function of least degree (in standard form) with integral coefficients.

$\pm 3,\ 4-3i,\ 4+3i$

Sum: 8
product: 25

$(4-3i)(4+3i)$

$16-9i^2$

25

$f(x) = (x-3)(x+3)$

$f(x) = (x^2-9)(x^2-8x+25)$

$$f(x) = x^4-8x^3+16x^2+72x-225$$

$$\begin{array}{r|l} & x^2-8x+25 \\ \hline x^2 & x^4-8x^3+25x^2 \\ -9 & -9x^2+72x-225 \end{array}$$

Write a polynomial function of least degree (in standard form) with integral coefficients.

$1-\sqrt{2},\quad 2-\sqrt{3}$

$1-\sqrt{2},\ 1+\sqrt{2}$

sum: 2

product: -1

(x^2-2x-1)

$2-\sqrt{3},\ 2+\sqrt{3}$

sum: 4

product: 1

(x^2-4x+1)

$f(x)=x^4-6x^3+8x^2+2x-1$