2.6: The Rational Zero Theorem

If $f(x) = a_n x^n + \dots + a_1 x + a_0$, has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

$f(x) = x^3 - 4x^2 - 6x + 8$

Constant Term: 8

Leading coefficient: 1

$p: \pm 1, \pm 2, \pm 4, \pm 8$

$q: \pm 1$

$\pm 1, \pm 2, \pm 4, \pm 8$

$g(x) = 2x^3 + 7x^2 - 4x + 8$

List the possible rational zeros.

constant term: 8

lead coeff: 2

p: $\pm 1, \pm 2, \pm 4, \pm 8$

q: $\pm 1, \pm 2$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2},$

$h(x) = 3x^3 + 4x^2 - 35x - 12$

List the possible rational zeros.

Constant term: -12

lead coeff: 3

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$q: \pm 1, \pm 3$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

Find the zeros of f(x)

$$f(x) = x^3 - 8x^2 + 11x + 20$$

Goal: find a zero!!!

constant: 20

lead coeff: 1

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

2 | 1 −8 11 20

2 −12 −2

1 −6 −1 X

4 | 1 −8 11 20

4 −16 −20

1 −4 −5 0

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

5, −1

4, 5, −1

Find the zeros of f(x)

$f(x) = x^3 - 4x^2 - 15x + 18$

possible rational zeros : $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

Goal: find a zero!!!

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

1, 6, -3

Homework:
Cross off Pg. 141
Add
Pg. 133: 25 - 29 odd

$f(x) = 3x^3 + 19x^2 + 4x - 12$

$$\begin{array}{r|cccc} -1 & 3 & 19 & 4 & -12 \\ & & -3 & -16 & 12 \\ \hline & 3 & 16 & -12 & 0 \end{array}$$

$3x^2 + 16x - 12 = 0$

$(3x - 2)(x + 6)$

$-1, -6, \frac{2}{3}$

33) $f(x) = 2x^3 - 10x^2 - 71x - 9$; 9

$$\begin{array}{r|rrrr} 9 & 2 & -10 & -71 & -9 \\ & & 18 & 72 & 9 \\ \hline & 2 & 8 & 1 & 0 \end{array}$$

$2x^2 + 8x + 1 = 0$

$$\frac{-4 \pm \sqrt{14}}{2}$$

$2\left(x^2 + 4x + \underline{4}\right) \underline{-8} + 1$

$2(x+2)^2 - 7 = 0$

$\sqrt{(x+2)^2} = \sqrt{\frac{7}{2}}\frac{\sqrt{2}}{\sqrt{2}}$

$x + 2 = \frac{\pm\sqrt{14}}{2}$

$x = -2 \pm \frac{\sqrt{14}}{2}$

41.) $P = -x^3 + 4x^2 + x$

$4 = -x^3 + 4x^2 + x$

$0 = -x^3 + 4x^2 + x - 4$

$0 = -x^2(x-4) + 1(x-4)$

$(-x^2+1)(x-4)$

$-x^2 + 1 = 0$

$x^2 = 1$

$x = \pm 1$

31.) $\boxed{7}$ | 10 −81 71 42

70 −77 −42

10 −11 −6 0

$10x^2 - 11x - 6 = 0$

$(5x + 2)(2x - 3) = 0$

5 2

2 −3

−15 4

$\boxed{-2/5, 3/2}$