

2.5 Apply the Remainder and Factor Theorems



"To show you how well I understand fractions,
I only did half of my homework."

HW:

2.5 Apply the Remainder and Factor Theorems

Polynomial Division Techniques

1. Dividing By A Monomial
2. Dividing By A Polynomial
 - Long Division
 - Synthetic Division

1. Dividing By A Monomial

ex: Divide.

$$\frac{7x^4 - 5x^2 + 14x}{21x^3}$$

$$\frac{7x^4}{21x^3} - \frac{5x^2}{21x^3} + \frac{14x}{21x^3}$$

$$\frac{x}{3} - \frac{5}{21x} + \frac{2}{3x^2}$$

2. Dividing By A Polynomial - Long Division

REVIEW: Divide.

$$130 + \frac{3}{4}$$

$$\begin{array}{r} 130\frac{3}{4} \\ 4 \overline{)523} \\ \underline{-4} \\ 12 \\ \underline{-12} \\ 3 \\ \underline{-3} \\ 0 \\ 3 \end{array}$$

$$\begin{array}{r} \div \\ X \\ \hline \end{array}$$

ex: Divide using long division.

a) $\frac{x^2 + 2x + 7}{x - 8}$

$$\begin{array}{r} \frac{x^2}{x} \\ x(x-8) \\ -(x^2-8x) \end{array}$$

$$\begin{array}{r} \underline{x-8} \overline{) x^2 + 2x + 7} \\ \underline{-x^2 + 8x} \\ 10x + 7 \\ \underline{-10x + 80} \\ 87 \end{array} \quad \begin{array}{r} \div \\ x \\ - \end{array}$$

$x + 10 + \frac{87}{x-8}$

ex: Divide using long division.

$$b) \frac{x^2 + 3x - 40}{x + 5}$$

$$\begin{array}{r} x+5 \overline{) x^2 + 3x - 40} \\ \underline{-x^2 + 5x} \\ -2x - 40 \\ \underline{+2x + 10} \\ -30 \end{array}$$

$$x - 2 + \frac{-30}{x+5}$$

$$-(x^2 + 5x)$$

same

$$x - 2 - \frac{30}{x+5}$$

2. Dividing By A Polynomial - Synthetic Division

ex: Divide using synthetic division.

$$a) \frac{x^2 + 2x + 7}{x - 8}$$

set den. = 0
 $x - 8 = 0$
 $x = 8$

$$\begin{array}{r|rrr} 8 & 1 & 2 & 7 \\ & & 8 & 80 \\ \hline & 1 & 10 & 87 \end{array}$$

remainder

$$x + 10 + \frac{87}{x - 8}$$

ex: Divide using synthetic division.

b) $\frac{x^2 + 3x - 40}{x + 5}$

$$\begin{array}{r|rrr} -5 & 1 & 3 & -40 \\ & & -5 & 10 \\ \hline & 1 & -2 & -30 \end{array}$$

$x - 2 + \frac{-30}{x + 5}$

ex: Divide using synthetic division.

$$c) \frac{x^2 - 4}{x - 1}$$

$$x + 1 + \frac{-3}{x - 1}$$

$$x + 1 - \frac{3}{x - 1}$$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -4 & \\ & & 1 & 1 & \\ \hline & 1 & 1 & -3 & \\ & \text{---} & \text{---} & \text{---} & \\ & |x + 1 + \frac{-3}{x - 1} & & & \end{array}$$

ex: Divide using any technique.

$$\text{a) } \frac{x^3 + 2x^2 + 2x + 9}{x^2 + 5}$$

$$x + 2 + \frac{-3x - 1}{x^2 + 5}$$

or

$$x + 2 - \frac{3x + 1}{x^2 + 5}$$

$$\begin{array}{r} x + 2 \\ \hline x^2 + 5 \overline{) x^3 + 2x^2 + 2x + 9} \\ \underline{-x^3 \quad + 5x} \\ 2x^2 - 3x + 9 \\ \underline{-2x^2 \quad + 10} \\ -3x - 1 \end{array}$$

ex: Divide using any technique.

$$b) \frac{x^3 + 7x^2 - x}{x+3}$$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & -1 & 0 \\ & & -3 & -12 & 39 \\ \hline & 1 & 4 & -13 & 39 \end{array}$$

1 degree
less than
numerator

$$x^2 + 4x - 13 + \frac{39}{x+3}$$

ex: Divide using any technique.

$$c) \frac{x^2 + x\sqrt{3} + 10}{x + \sqrt{3}}$$

ex: Find c if

$$\frac{x^3 - 2x^2 + cx + 4}{x - 3} = x^2 + x + 3 + \frac{13}{x - 3}$$

Review

ex: Perform the indicated operation.

$$(x - 5)(x^3 + 5x^2 + 2x - 1)$$

Review

ex: Classify the polynomial product in the last example.

ex: Identify the degree and leading coefficient in the last example.

Review

ex: Simplify.

$$\frac{3x^{-7}y^5z^0}{(3xy^{-1}z^5)^{-2}}$$