2.5: Dividing Polynomials with Long Division and Synthetic Division

$$
\begin{array}{r|rrr}
 & 1 & 4 & 5 \\
\hline
3 & 4 & 3 & 7 \\
 & -3 & \downarrow & \\
\hline
 & 1 & 3 & \\
 & -1 & 2 & \downarrow \\
\hline
 & & 1 & 7 \\
 & & -1 & 5 \\
\hline
 & & & 2
\end{array}
$$

$145\frac{2}{3}$

$145 + \frac{2}{3}$

Divide using long division

1) $(3x^2 - 11x - 26) \div (x - 5)$

$$
\begin{array}{r}
3x + 4 \\
x - 5 \overline{\big) 3x^2 - 11x - 26} \\
-3x^2 + 15x \\
\hline
4x - 26 \\
-4x + 20 \\
\hline
-6
\end{array}
$$

$$\frac{3x^2}{x}$$

$$\boxed{3x + 4 + \frac{-6}{x-5}}$$

or

$$3x + 4 - \frac{6}{x-5}$$

1a.) $(3x^2 - 4x - 4) \div (x - 4)$

$$
\begin{array}{r}
3x + 8 \\
x - 4 \overline{\big) 3x^2 - 4x - 4} \\
-3x^2 + 12x \\
\hline
8x - 4 \\
-8x + 32 \\
\hline
28
\end{array}
$$

$$3x + 8 + \frac{28}{x - 4}$$

$$3x + 8 + 28/(x - 4)$$

A remainder means that (x - 4) is NOT a factor of ($3x^2$ - 4x - 4)

$$(x^2 + 4x + 3) \div (x + 3)$$

$$
\begin{array}{r}
x + 1 \\
x+3 \overline{\big) x^2 + 4x + 3} \\
-\,x^2 + {}^{-}3x \\
\hline
x + 3 \\
-\,x + {}^{-}3 \\
\hline
0
\end{array}
$$

A remainder zero means that (x +3) is a factor of $x^2 + 4x + 3$

Divide using long division

2) $(7x^4 + 11x^2 + 7x + 5) \div (x^2 + 1)$

$$
\begin{array}{r}
7x^2 + 4 \\
x^2 + 0 + 1 \overline{\big) 7x^4 + 0 + 11x^2 + 7x + 5} \\
-7x^4 + 0 - 7x^2 \\
\hline
4\cancel{x^2} + 7x + 5 \\
-4\cancel{x^2} + 0 - 4 \\
\hline
7x + 1
\end{array}
$$

$$\boxed{7x^2 + 4 + \frac{7x+1}{x^2+1}}$$

Divide using synthetic division.

3) $(4x^2 - 13x + 10) \div (x - 2)$

corner numbe

$x - 2 = 0$

$x = 2$

$$\begin{array}{r|rrr} 2 & 4 & -13 & 10 \\ & \downarrow & 8 & -10 \\ \hline & 4 & -5 & 0 \end{array}$$

$$\boxed{4x - 5}$$

Divide using synthetic division.

4) $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

corner number: -5

-5	1	4	0	16	-35
		-5	5	-25	45
	1	-1	5	-9	$\boxed{10}$

remainder

$$\boxed{1x^3 - 1x^2 + 5x - 9 + \frac{10}{x+5}}$$

Divide using synthetic division.

5.) $(x^3 - 4x + 6) \div (x+3)$

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -4 & 6 \\ & & -3 & 9 & -15 \\ \hline & 1 & -3 & 5 & \boxed{-9} \end{array}$$

$$\boxed{x^2 - 3x + 5 + \frac{-9}{x+3}}$$