

2.5 Apply the Remainder and Factor Theorems

Polynomial Division Techniques

1. Dividing By A Monomial
2. Dividing By A Polynomial
 - Long Division
 - Synthetic Division

1. Dividing By A Monomial

ex: Divide.

$$\frac{7x^4 - 5x^2 + 14x}{21x^3}$$

$$\frac{7x^4}{21x^3} - \frac{5x^2}{21x^3} + \frac{14x}{21x^3}$$
$$\frac{x}{3} - \frac{5}{21x} + \frac{2}{3x^2}$$

2. Dividing By A Polynomial - Long Division

REVIEW: Divide.

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$$\begin{array}{r} 130\frac{3}{4} \\ 4 \overline{) 523} \\ \underline{-4} \\ 12 \\ \underline{-12} \\ 03 \\ \underline{0} \\ 3 \end{array}$$

ex: Divide using long division.

a) $\frac{x^2 + 2x + 7}{x - 8}$

$\begin{array}{r} \div \\ \times \\ - \end{array}$

$$\begin{array}{r} \underline{x-8} \overline{) x^2 + 2x + 7} \\ \underline{-(x^2 - 8x)} \\ 10x + 7 \\ \underline{-(10x - 80)} \\ 87 \end{array}$$

$$\frac{10x}{x}$$

$$x + 10 + \frac{87}{x-8}$$

ex: Divide using long division.

$$\text{b) } \frac{x^2 + 3x - 40}{x + 5}$$

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$$\begin{array}{r} x+5 \overline{) x^2 + 3x - 40} \\ \underline{-(x^2 + 5x)} \\ -2x - 40 \\ \underline{-(-2x - 10)} \\ -30 \end{array}$$

$x - 2 + \frac{-30}{x+5}$

2. Dividing By A Polynomial - Synthetic Division

ex: Divide using synthetic division.

a) $\frac{x^2 + 2x + 7}{x - 8}$

$x - 8 = 0$
 $x = 8$

$$\begin{array}{r|rrr} 8 & 1 & 2 & 7 \\ & & 8 & 80 \\ \hline & 1 & 10 & 87 \end{array}$$

$x + 10 + \frac{87}{x - 8}$

ex: Divide using synthetic division.

$$\text{b) } \frac{x^2 + 3x - 40}{x + 5}$$

$$\begin{array}{r|rrr} -5 & 1 & 3 & -40 \\ & & -5 & 10 \\ \hline & 1 & -2 & -30 \end{array}$$

$x - 2 - \frac{30}{x + 5}$

ex: Divide using synthetic division.

$$c) \frac{x^2 - 4}{x - 1}$$

$$\frac{x^2}{x} = x$$

$$\begin{array}{r|rrr} 1 & 1 & 0 & -4 \\ & & 1 & 1 \\ \hline & 1 & 1 & -3 \end{array}$$

$$x + 1 - \frac{3}{x - 1}$$

ex: Divide using any technique.

$$\text{a) } \frac{x^3 + 2x^2 + 2x + 9}{x^2 + 5}$$

$$\begin{array}{r} x + 2 + \frac{-3x-1}{x^2+5} \\ \underline{x^2 + 5} \overline{) x^3 + 2x^2 + 2x + 9} \\ - (x^3 \quad + 5x) \\ \hline 2x^2 - 3x + 9 \\ - (2x^2 \quad + 10) \\ \hline -3x - 1 \end{array}$$

ex: Divide using any technique.

$$b) \frac{x^3 + 7x^2 - x}{x+3}$$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & -1 & 0 \\ & & -3 & -12 & 39 \\ \hline & 1 & 4 & -13 & 39 \\ & & & & \frac{39}{x+3} \end{array}$$
$$x^2 + 4x - 13 + \frac{39}{x+3}$$

ex: Divide using any technique.

$$c) \frac{x^2 + x\sqrt{3} + 10}{x + \sqrt{3}}$$

ex: Find c if

$$\frac{x^3 - 2x^2 + cx + 4}{x - 3} = x^2 + x + 3 + \frac{13}{x - 3}$$