

2.5: Dividing Polynomials with Long Division and Synthetic Division

$$\begin{array}{r} & 1 & 5 & 3 \\ \hline 3 &) & 4 & 6 & 1 \\ & -3 & & & \\ \hline & 1 & 6 & & \\ & -1 & 5 & & \\ \hline & & 1 & & \\ & & -9 & & \\ \hline & & & 2 & \end{array}$$

A diagram illustrating synthetic division. A blue-outlined rectangle contains the calculation $153\frac{2}{3} \div 3$. The result is shown as $153 + \frac{2}{3}$.

Steps

- 1) Divide
- 2) Multiply
- 3) Subtract

Divide using long division

1) $(3x^2 - 11x - 26) \div (x - 5)$

$$\begin{array}{r} 3x + 4 \\ x - 5 \overline{)3x^2 - 11x - 26} \\ -3x^2 + 15x \quad \downarrow \\ \hline 4x - 26 \\ -4x + 20 \\ \hline -6 \end{array}$$

$$3x + 4 + \frac{-6}{x-5}$$

Divide: $\frac{3x^2}{x} = 3x$

Multiply $3x(x-5)$

Subtract: change signs

$$2) (x^2 - x - 12) \div (x - 4) = x + 3$$

$$\begin{array}{r} x+3 \\ \hline x-4 \overline{)x^2 - x - 12} \\ -x^2 + 4x \\ \hline -3x - 12 \\ -3x + 12 \\ \hline 0 \end{array}$$

A remainder zero means that $(x - 4)$ is a factor of $x^2 - x - 12$

*Similar: 12 divided by 4 has zero remainder,
4 is a factor of 12*

Divide using long division

$$3) (7x^4 + 11x^2 + 7x + 5) \div (x^2 + 1)$$

$$\begin{array}{r} 7x^2 + 4 \\ \hline x^2 + 0 + 1 \longdiv{7x^4 + 0 + 11x^2 + 7x + 5} \\ -7x^4 + 0 + 7x^2 \\ \hline 4x^2 + 7x + 5 \\ -4x^2 + 4 \\ \hline 7x + 1 \end{array}$$

$$\boxed{7x^2 + 4 + \frac{7x+1}{x^2+1}}$$

Divide using synthetic division.

4) $(4x^2 - 13x + 10) \div (x - 2)$

$$\begin{aligned}x - 2 &= 0 \\x &= 2\end{aligned}$$

$$\begin{array}{r|rrr}2 & 4 & -13 & 10 \\ & 8 & 10 & \\ \hline & 4 & -5 & 0\end{array}$$

$$\boxed{4x - 5}$$

*These values are
coefficients of the
answer*

Divide using synthetic division.

5) $(x^4 + 4x^3 + 16x - 35) \div (x + 5)$

down one
synthetic

-5	1	4	0	16	-35
	-5	5	-25	45	
	1	-1	5	-9	10
	$x^3 - 1x^2 + 5x - 9 + \frac{10}{x+5}$				

$$(2x^3 - x^2 - 8) \div (x-1)$$

$$\begin{array}{r} 1 \\ | \begin{array}{cccc} 2 & -1 & 0 & -8 \\ 2 & 1 & 1 & \\ \hline 2 & 1 & 1 & -7 \end{array} \end{array}$$

$$2x^2 + 1x + 1 + \frac{-7}{x-1} \text{ or}$$

$$2x^2 + x + 1 - \frac{7}{x-1}$$

2.5 Apply the Remainder and Factor Theorems Cont.

~~2.6 Find Rational Zeros~~

The Factor Theorem

A polynomial $g(x)$ is a factor of $f(x)$ if...

1. $\frac{f(x)}{g(x)}$ has a remainder of 0.

2. k is a zero of $g(x)$ and $f(k) = \underline{\hspace{2cm}0}$.

ex: Is $g(x)$ a factor of $f(x)$?

a) $g(x) = x - 5$, $f(x) = x^3 - 7x^2 + 7x + 15$

$$\begin{array}{r} 5 \Big| 1 \ -7 \ 7 \ 15 \\ \underline{-5 \ -10 \ -15} \\ 1 \ -2 \ -3 \ 0 \end{array}$$

Yes;
0 remainder

ex: Factor $f(x)$ completely given one of its factors.

a) $f(x) = 15x^3 + x^2 - 22x - 8$; $x + 1$

$$\begin{array}{r} -1 \Big| 15 \ 1 \ -22 \ -8 \\ \quad -15 \ 14 \ 8 \\ \hline 15 \ -14 \ -8 \ 0 \end{array}$$

$$f(x) = (x+1)(5x+2)(3x-4)$$

$$\begin{aligned} &15x^2 - 14x - 8 \\ &(5x+2)(3x-4) \end{aligned}$$

ex: Factor $f(x)$ completely given one of its factors.

b) $f(x) = x^3 - 7x^2 + 7x + 15$; $x - 3$

$$\begin{array}{r} 3 \Big| 1 \ -7 \ 7 \ 15 \\ \quad 3 \ -12 \ -15 \\ \hline \quad 1 \ -4 \ -5 \end{array}$$

$f(x) = (x+1)(x-3)(x-5)$

$$x^2 - 4x - 5$$
$$(x-5)(x+1)$$

ex: Find the zeros of $f(x)$ given one of its zeros.

$$f(x) = x^3 + 6x^2 + 9x + 4; \quad -4$$

$$\begin{array}{r} -4 \\ \hline 1 & 6 & 9 & 4 \\ -4 & -8 & -4 \\ \hline 1 & 2 & 1 & 0 \end{array}$$

$$x^2 + 2x + 1 = 0$$
$$(x+1)^2 = 0$$

zeros:
 $x = -4, -1$
mult. of 2

Simplify using laws of exponents

Categorize polynomials by degree and by # of terms

Add, subtract, and multiply polynomials

Factor completely (including sum/difference of cubes)

Solve equations by factoring

Dividing polynomials using long division/synthetic

Factor/solve given a factor or a zero

$$(8x^2 + 34x - 1) \div (4x - 1)$$

$2x + 9$

$4x - 1 \overline{)8x^2 + 34x - 1}$

$- 8x^2 + 2x$

$ - 36x - 1$

$ + 9$

Long Division

$2x + 9 + \frac{8}{4x - 1}$