9.) $x^2+2x-4 \overline{)5x^4-2x^3-7x^2+0x-39}$

$5x^2-12x+37$

$-(5x^4+10x^3-20x^2)$

$-12x^3+13x^2+0x$

$-(-12x^3-24x^2+48x)$

$37x^2-48x-39$

$-(37x^2+74x-148)$

$-122x+109$

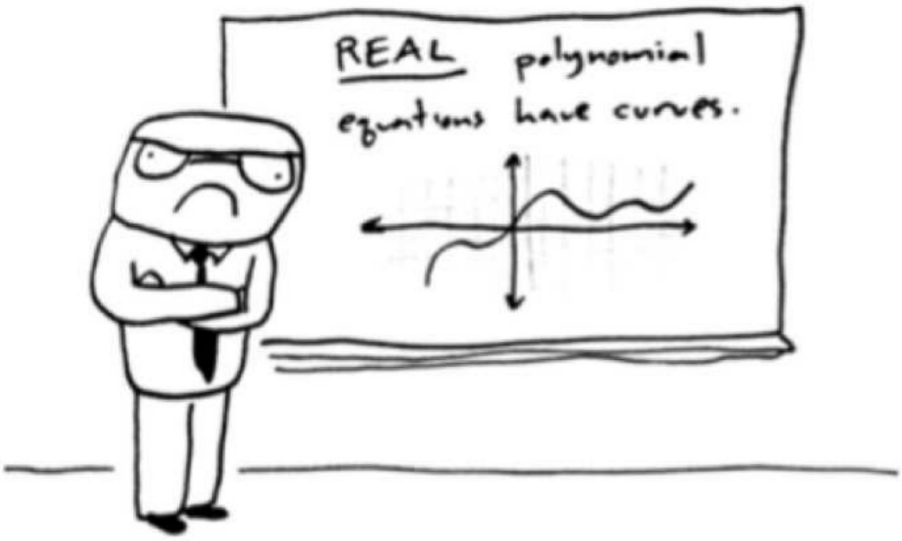
÷

X

—

$$5x^2-12x+37+\frac{-122x+109}{x^2+2x-4}$$

2.5 Apply the Remainder and Factor Theorems Cont.
2.6 Find Rational Zeros
2.7 Finding All Zeros



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HW:

Set A : 3, 4, 5

p. 124 : 21, 29, 31

WS : 1, 2, 5

↑ finding zeros of poly.

The Factor Theorem

A polynomial $g(x)$ is a factor of $f(x)$ if...

1. $\frac{f(x)}{g(x)}$ has a remainder of __0__.

2. k is a zero of $g(x)$ and $f(k) =$ __0__.

ex: Is $g(x)$ a factor of $f(x)$? Yes!

a) $g(x) = x - 5, \quad f(x) = x^3 - 7x^2 + 7x + 15$

5	1	-7	7	15
		5	-10	-15
	1	-2	-3	0

ex: Is $g(x)$ a factor of $f(x)$? No!

b) $g(x) = x + 7, \quad f(x) = x^2 - 9$

$$\begin{array}{c|ccc} -7 & 1 & 0 & -9 \\ & & -7 & 49 \\ \hline & 1 & -7 & 40 \end{array}$$

ex: Factor $f(x)$ completely given one of its factors.

a) $f(x) = 15x^3 + x^2 - 22x - 8; \quad x+1$

$$\begin{array}{r|cccc} -1 & 15 & 1 & -22 & -8 \\ & & -15 & 14 & 8 \\ \hline & 15 & -14 & -8 & 0 \end{array}$$

$15x^2 - 14x - 8 = 0$

$(3x - 4)(5x + 2) = 0$

$f(x) = (x+1)(3x-4)(5x+2)$

ex: Factor $f(x)$ completely given one of its factors.

b) $f(x) = x^3 - 7x^2 + 7x + 15; \qquad x - 3$

$f(x) = (x-3)(x-5)(x+1)$

ex: Find the zeros of $f(x)$ given one of its zeros.

$$f(x) = x^3 + 6x^2 + 9x + 4; \qquad -4$$

$$\begin{array}{r|cccc} -4 & 1 & 6 & 9 & 4 \\ & & -4 & -8 & -4 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$x^2 + 2x + 1 = 0$

$(x+1)(x+1) = 0$

$x = -4, -1$, mult. of 2

The Rational Zero Theorem

If $f(x)$ is a polynomial then every rational zero of $f(x)$ comes in the form of...

$$\frac{p}{q} = \frac{\text{factor of constant term}}{\text{factor of leading coefficient}}$$

ex: List the possible rational zeros.

a) $f(x) = 2x^3 - 7x^2 + 9$

(q = 2, p = 9)

$p: \pm 1, \pm 3, \pm 9$

$q: \pm 1, \pm 2$

$\frac{p}{q}: \pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

ex: List the possible rational zeros.

b) $f(x) = 4x^4 - x^3 - 7x^2 + 4x - 2$

q p

If a factor repeats, don't write it twice.

p : $\pm 1, \pm 2$

q : $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2$

Finding ALL Zeros

Before finding all zeros consider whether the polynomial factors

- if YES, then factor
- if NO, then use the rational zero thm. (p/q)

*If the polynomial is QUADRATIC,
then try factoring, CTS, or Quad form.

ex: Determine the best method for finding the zeros of the given polynomial

a) $f(x) = 10x^3 - 17x^2 - 7x + 2$

$x^2(10x - 17) - (7x - 2)$

not factorable : Rat. Zero Thm.

b) $f(x) = 16x^4 - 54x$

$2x(8x^3 - 27)$

factoring

ex: Find all zeros.

a) $f(x) = x^3 + 7x^2 + 15x + 9$

$p: \pm 1, \pm 3, \pm 9$

$q: \pm 1$

$\frac{p}{q}: \pm 1, \pm 3, \pm 9$

$$\begin{array}{c|cccc} -1 & 1 & 7 & 15 & 9 \\ & & -1 & -6 & -9 \\ \hline & 1 & 6 & 9 & 0 \end{array}$$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$\boxed{x = -1, -3 \text{ (mult. of 2)}}$

ex: Find all zeros.

b) $f(x) = x^3 - 9x^2 + 21x - 4$

$\frac{P}{q}: \pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} 4 & 1 & -9 & 21 & -4 \\ & & 4 & -20 & 4 \\ \hline & 1 & -5 & 1 & 0 \end{array}$$

$x^2 - 5x + 1 = 0$

$x = \frac{5 \pm \sqrt{21}}{2}, 4$

~~-4~~

$$\begin{array}{r|rrrr} & 1 & -9 & 21 & -4 \\ & & -4 & 52 & \\ \hline & 1 & -13 & 73 & \end{array}$$

ex: Find all zeros.

c) $f(x) = x^3 - 3x^2 + 4x - 12$

$0 = x^2(x-3) + 4(x-3)$

$0 = (x^2+4)(x-3)$

$x = 3, \pm 2i$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

ex: Find all zeros.

d) $f(x) = x^4 - 16x^2 - 40x - 25$

$\frac{P}{q}: \pm 1, \pm 5, \pm 25$

$5 \mid 1 \quad 0 \quad -16 \quad -40 \quad -25$

$\quad\quad 5 \quad 25 \quad 45 \quad 25$

$1 \quad 5 \quad 9 \quad 5 \quad 0$

$x^3 + 5x^2 + 9x + 5 = 0$

$-1 \mid 1 \quad 5 \quad 9 \quad 5$

$\quad\quad -1 \quad -4 \quad -5$

$1 \quad 4 \quad 5 \quad 0$

$x^2 + 4x + 5 = 0$

$x^2 + 4x \underline{+4} \underline{-4} + 5 = 0$

$(x+2)^2 = \sqrt{-1}$

$|x+2| = i \; ; \; x = -2 \pm i$

$x = 5, -1, -2 \pm i$

Review

ex: If $f(3)=0$, which statement(s) about f(x) are true?

a) $x+3$ is a factor of f(x)

b) $x-3$ is a factor of f(x)

c) -3 is a root of f(x)

d) $\frac{f(x)}{x-3}$ has a remainder of zero.