27.) $32z^5 - 2z$

$2z(16z^4 - 1)$

$2z(4z^2 + 1)(4z^2 - 1)$

$2z(4z^2 + 1)(2z + 1)(2z - 1)$

54) $x^3y^6 - 27$ $a^3 - b^3$

$(xy^2 - 3)(x^2y^4 + 3xy^2 + 9)$ $a = xy^2$
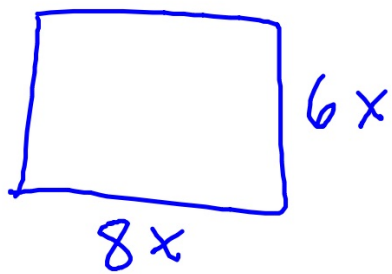 $b = 3$

56.) $x^{2n} - 2x^n + 1$ $x^2 - 2x + 1$
      $(x^n - 1)(x^n - 1)$

$6x$

$8x$

$h = x$

$48x^3$

2.4: Solving Polynomial Equations

Some solutions will be real and others imaginary. Also, we will identify the multiplicity of the equation.

The degree of the equation will determine the number of solutions. Example: degree 5 will be 5 solutions

What is multiplicity?

How many times a particular number is a solution for a given equation. For example, in the polynomial equation $(x - 3)^4(x - 5)(x - 8)^2 = 0$:

There are 7 solutions but with varying multiplicity:

x = 3 (mult of 4)
x = 5 (mult of 1)*
x = 8 (mult of 2)

*You do not have to specify when the multiplicty is 1.

Directions on HW:

a) How many solutions does the equation have?

b) Solve by factoring. State the multiplicity of any repeated solutions

c) Describe the solutions: How many are real? Imaginary?

③ $x^3 - 2x^2 + x = 0$

a.) 3

b.) $x(x^2 - 2x + 1) = 0$

$x(x-1)^2 = 0$

$x = 0, \ x = 1$

Mult. of 2

c.) 3 reals

0 imaginary

$\text{(9)} \quad 5x^6 = 9x^4 - 4x^2$

a.) 6

b.) $5x^6 - 9x^4 + 4x^2 = 0$

$x^2 \left(5x^4 - 9x^2 + 4 \right) = 0$

$x^2 \left(5x^2 - 4 \right) \left(x^2 - 1 \right) = 0$

$x^2 \left(5x^2 - 4 \right) \left(x + 1 \right) \left(x - 1 \right) = 0$

$x = 0 \qquad 5x^2 - 4 = 0 \qquad x = \pm \frac{2\sqrt{5}}{5} \qquad x = 1, \ x = -1$

mut. of $\qquad \qquad x^2 = \frac{4}{5}$

2

④ $12x^3 + 4x^2 + 3x + 1 = 0$

a.) 3

b.) $4x^2(3x+1) + 1(3x+1) = 0$

$(4x^2+1)(3x+1) = 0$

$4x^2+1 = 0 \qquad \downarrow$

$\sqrt{x^2} = \sqrt{\frac{-1}{4}} \qquad x = \frac{-1}{3}$

$x = \pm\frac{1}{2}i$

c.) 1 real, 2 imag

(17) $x^6 - 64 = 0$

a.) 6

c.) 2 reals
4 imaginary

b.) $(x^3 + 8)(x^3 - 8) = 0$

$(x+2)(x^2-2x+4)(x-2)(x^2+2x+4) = 0$

$\boxed{x=-2}$ $(x^2-2x+1)+4-1$ $\boxed{x=2}$ $x = \dfrac{-2 \pm \sqrt{4-4(1)(4)}}{2(1)}$

$(x-1)^2+3 = 0$

$\boxed{1 \pm i\sqrt{3}}$

$x = \dfrac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$

$\dfrac{-2}{2} \pm \dfrac{2i\sqrt{3}}{2}$

Write a polynomial equation in standard form with integral coefficients with the given roots (solutions).

a) $2, 3$ mult. of 2

$(x-2)(x-3)^2 = 0$

$(x-2)(x^2-6x+9) = 0$

$x^3-8x^2+21x-18 = 0$

$$\begin{array}{r} x^2 \quad -6x \quad 9 \\ \hline x \quad | \; x^3 - 6x^2 \quad 9x \\ -2 \; | \; -2x^2 \quad 12x \quad -18 \end{array}$$

b.) $\sqrt{6}, -\sqrt{6}, \sqrt{3}, -\sqrt{3}$

$(x-\sqrt{6})(x+\sqrt{6})(x-\sqrt{3})(x+\sqrt{3})$

$(x^2-6)(x^2-3)$

$x^4-9x^2+18=0$

15) $18x^3 = 50x$

a.) 3

b.) $18x^3 - 50x = 0$

$2x(9x^2 - 25) = 0$

$2x(3x+5)(3x-5) = 0$

$x = 0, -\frac{5}{3}, \frac{5}{3}$

c.) all real