45.) $\left(a^5 b^4\right)^2 = a^{14} b^{-1} \cdot a^{-4} \cdot b^9$

$\downarrow$

$a^{10} b^8$

34.) $\dfrac{y''}{4z^3} \cdot \dfrac{8z^7}{y^7}$

$\dfrac{2\,\cancel{8}y''\cancel{z^7}\,y^4 z^4}{\cancel{4}\cancel{y^7}\cancel{z^3}}$

$2y^4 z^4$

35.) $$\dfrac{x^2 y^2 x^4}{3y^2 y^3}$$

$$\dfrac{x^6 y^2}{3y^5 y^{-2}} = \dfrac{1x^6}{3y^3}$$

25.) $(2^2 y^3)^5$

$2^5 = 32$

$2^{10} y^{15}$

$1024 y^{15}$

30.) $$\frac{x^{-1} y^{2}}{x^{2} y^{-1}} = \frac{y^{2} y^{1}}{x^{2} x^{1}}$$

$$= \frac{y^{3}}{x^{3}}$$

33.)

$$\frac{2 a^3 b^2}{3 a^5 b^4}$$

$$a^{-2}$$

$$b^{-2}$$

$$\frac{2}{3 a^2 b^2}$$

$$\frac{\cancel{a a a}}{\cancel{a a a a a}}$$
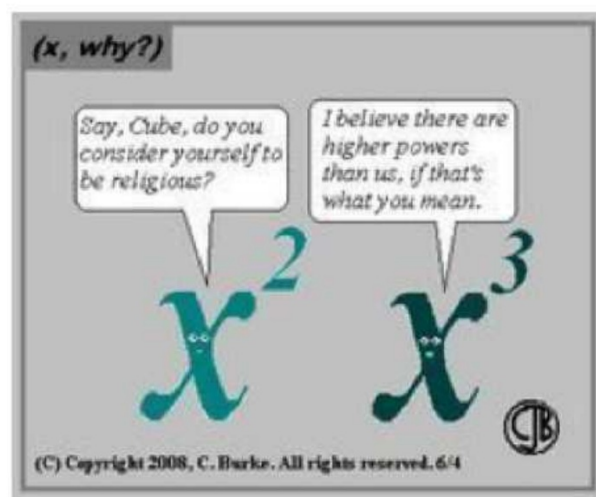
$$\frac{a^{-7}}{b^{-4}} = \frac{b^4}{a^7}$$

43.) $x^{15} y^{12} z^8 = x^4 y^7 z^{11} \cdot \dfrac{x^{11} y^5 z^{-3}}{}$

$x^4 \cdot x^{11}$

x^{15}

2.2 Evaluating Polynomial Functions
2.3 Add, Subtract and Multiply Polynomials

HW:

Monomial - a number, a variable or a product of numbers and variables

$$3 \qquad 7x \qquad 3x^2y^5$$

Polynomial - an expression involving one or more monomials

$$3x - 1 \qquad 5x^2 - 4x^2 + 3x - 5$$

$$x^2 + 4x - 3$$

Characteristics of Polynomials
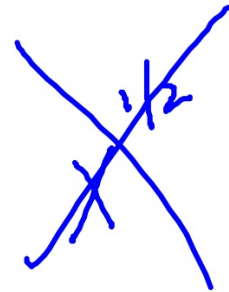
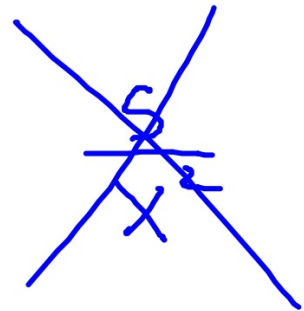1. variables have whole exponents

$$x^5, \quad x^0, \quad 3x,$$

$$x^{1/2} \quad \text{(crossed out)}$$

2. real coefficients

$$3x, \quad \sqrt{7}\,x^5$$

3. no division by variables

$$\frac{x}{3} = \frac{1}{3}x$$

$$\frac{5}{x^2} \quad \text{(crossed out)}$$

ex: Determine whether the expression represents a polynomial.

a) $2x^2 - 4x + \dfrac{1}{7}$ Y

b) 0 Y

c) $\dfrac{5}{x^2}$ N

d) $\dfrac{x^2}{5}$ Y

ex: Determine whether the expression represents a
polynomial.

e) $\sqrt{x} = x^{1/2}$ No

f) $\pi x + i^2$ Y

$\pi x - 1$

g) $3 - ix$ N

h) $\dfrac{x+1}{x+5}$ N

ex: Determine whether the expression represents a polynomial.

i) $-3x^3 + \dfrac{1}{2}x^4$ Y

j) $3^{1/2} = \sqrt{3}$

Y

Classifying Polynomials

1. (Degree) – largest exponent

Degree	Type
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic
≥ 6	n^{th} degree polynomial

Classifying Polynomials

2. Number of terms

Number of Terms	Type
1	monomial
2	binomial
3	trinomial
≥ 4	polynomial

ex: Classify the polynomial by the degree and number of terms.

a) $4x - 27x^2 + 3$ quadratic trinomial
↑

b) $\pi x + \pi^2$ linear binomial
↑

ex: Classify the polynomial by the degree and number of terms.

c) $17,000^4$ constant monomial

d) $(x-17)^2$ Quadratic trinomial
$$x^2 - 34x + 289$$

ex: Classify the polynomial by the degree and number of terms.

e) $5x^6 + 2x^3 + 4x - 5$

↑

6th degree polynomial

<u>Standard Form of a Polynomial</u> - a polynomial is in standard form when the terms' exponents are in descending order.

ex: Write the polynomial in standard form.

$$1 + 2x - 3x^4$$

$$-3x^4 + 2x + 1$$

Leading Coefficient - the coefficient of the term that defines the degree

$2x - 7$

2

$1 - 3x + 4X^3$

4

ex: Identify the leading coefficient.

$$x - \frac{3x^4}{5} + 10$$

$-\dfrac{3}{5}$

Evaluating Polynomials

There are two ways to evaluate polynomial functions:

      1. direct substitution

      2. synthetic substitution

<u>Direct Substitution</u> (i.e. "PLUG IN")

ex: Find the indicated polynomial value using direct substitution.

a) $f(x) = x^2 - 5x + 2,$ $f(13) = ?$

$f(13) = 13^2 - 5(13) + 2$

$ = 169 - 65 + 2$

$ = 104 + 2$

$ = 106$

ex: Find the indicated polynomial value using direct substitution.

b) $g(x) = x^3 + 4x^2 - 1,$ $g(6) = ?$

$$g(6) = 6^3 + 4(6)^2 - 1$$
$$= 216 + 144 - 1$$
$$= 360 - 1$$
$$= 359$$

Synthetic Substitution - substitution using a chart of coefficients

*Before using synthetic substitution,
- the polynomial must be in standard form
- consider if all terms are present

ex: Find the indicated polynomial value using synthetic substitution.

a) $f(x) = x^2 - 5x + 2,$ $f(13) = ?$

$$
\begin{array}{r|rrr}
13 & 1 & -5 & 2 \\
  &   & 13 & 104 \\
\hline
  & 1 & 8 & \boxed{106}
\end{array}
$$

ex: Find the indicated polynomial value using synthetic substitution.

b) $g(x) = x^3 + 4x^2 - 1,$ $g(6) = ?$

$$
\begin{array}{r|rrrr}
6 & 1 & 4 & 0 & -1 \\
 & \downarrow & 6 & 60 & 360 \\
\hline
 & 1 & 10 & 60 & 359 \\
\end{array}
$$

ex: Find the indicated polynomial value using synthetic substitution.

c) $m(x) = 5x^4 + 2x - 8, \qquad m(-2) = ?$

$$
\begin{array}{r|rrrrr}
-2 & 5 & 0 & 0 & 2 & 0 \\
   &   & -10 & 20 & -40 & 76 \\
\hline
   & 5 & -10 & 20 & -38 & \boxed{76}
\end{array}
$$

ex: If $f(x) = 3x^2 + bx - 7$ and $f(2) = 15$ find the value of b.

$15 = 3(2)^2 + 2b - 7$

$15 = 12 + 2b - 7$

$10 = 2b$

$5 = b$

$$
\begin{array}{c|ccc}
2 & 3 & b & -7 \\
  & \downarrow & 6 & 2b+12 \\
\hline
  & 3 & b+6 & 15
\end{array}
$$

$-7 + 2b + 12 = 15$

$\longrightarrow b = 5$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

a) $a(x) + b(x)$

$$-5 + 5x^4 + 2 = 5x^4 - 3$$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

b) $b(x) - c(x)$

$\left(5x^4 + 2\right) - \left(5x^2 + 4x - 3\right)$

$5x^4 + 2 - 5x^2 - 4x + 3$

$5x^4 - 5x^2 - 4x + 5$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation. Standard form

c) $d(x) - 5b(x)$

$(2x-1) - 5(5x^4 + 2)$

$2x - 1 - 25x^4 - 10$

$-25x^4 + 2x - 11$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

d) $a(x)b(x)d(x)$

$-5(5x^4 + 2)(2x - 1)$

$-5(10x^5 - 5x^4 + 4x - 2)$

$-50x^5 + 25x^4 - 20x + 10$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

e) $[d(x)]^2$ $(2x-1)^2$

$$= 4x^2 - 4x + 1$$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation.

f) $b(x)c(x) = (5x^4 + 2)(5x^2 + 4x - 3)$

$= 25x^6 + 20x^5 - 15x^4 + 10x^2 + 8x - 6$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

9) $c(x)[d(x)]^2$

$(5x^2 + 4x - 3)(4x^2 - 4x + 1)$

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad\qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation.

g) Box Method: $c(x)\left[d(x)\right]^2$

$$20x^4 - 4x^3 - 23x^2 + 16x - 3$$

	$5x^2 +$	$4x -$	3
$4x^2$	$20x^4$	$16x^3$	$-12x^2$
$-4x$	$-20x^3$	$-16x^2$	$12x$
1	$5x^2$	$4x$	-3

$$\frac{y^5}{459x^5z^6}$$

$$\frac{\left(3x^4y^{-5}\right)^{-2}z^0}{51x^{-3}y^5z^6} = \frac{3^{-2}x^{-8}y^{10}x^3}{51y^5z^6}$$

$$\frac{y^{10}x^3}{9.51x^8y^5z^6} = \frac{y^5}{459x^5z^6}$$