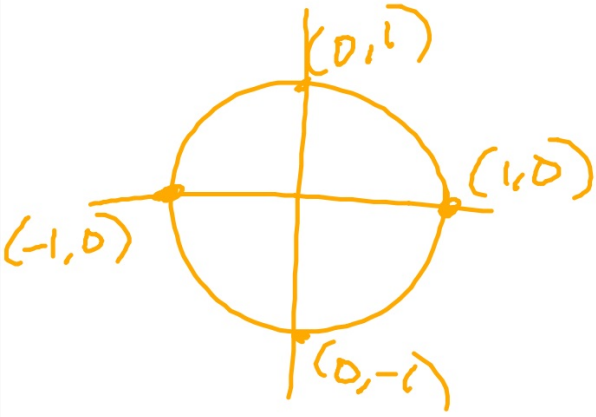
10.1 Graphing Sine and Cosine Functions

We can use the unit circle to graph y = sinx and y = cosx

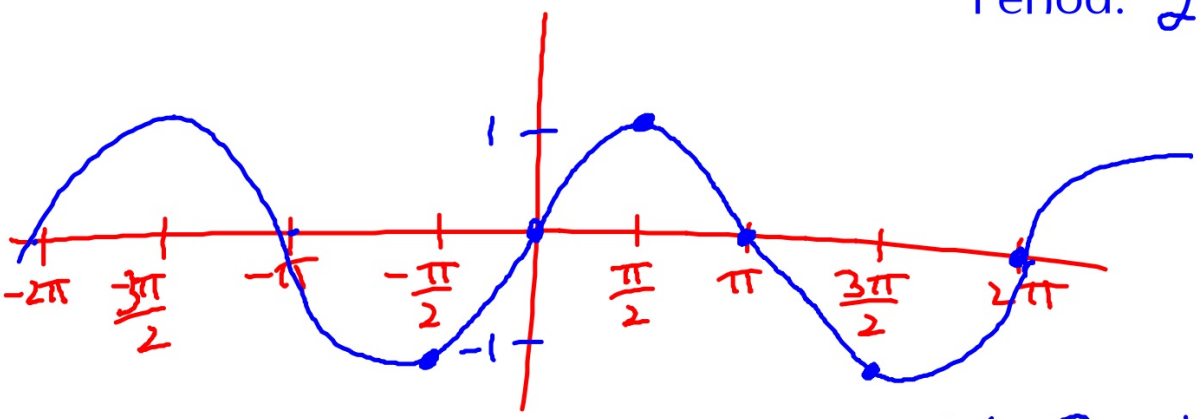


$\sin\frac{\pi}{2} = 1 \quad \left(\frac{\pi}{2}, 1\right)$

$\cos\pi = -1 \quad (\pi, -1)$

$y = \sin x$

Period: 2π



D: $\{x \mid x \in R\}$

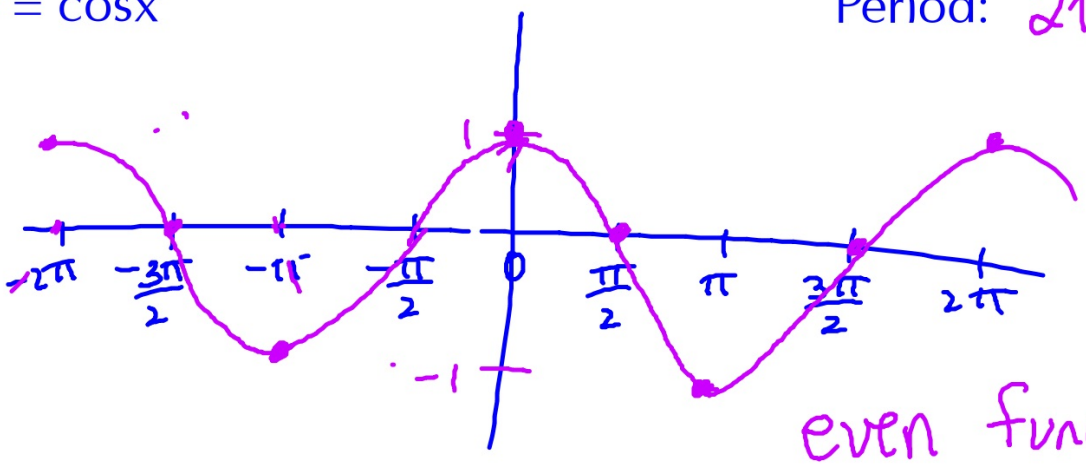
R: $\{y \mid -1 \leq y \leq 1\}$

odd function

symm. with origin

y = cosx

Period: 2π



even function

symm. w/ y-axis

D: $\{x \mid x \in R\}$

R: $\{y \mid -1 \leq y \leq 1\}$

Transformations

$y = 3\sin 2x - 1$

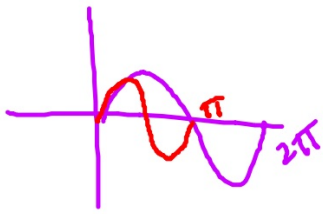
y = asinb(x - c) + d or y = acosb(x - c) + d

a: Amplitude | a | $a = 3$ Amp = 3

b: This value determines the period of the function

period = 2pi/b Period $= \frac{2\pi}{2} = \pi$

$b = 2$



c: phase shift (aka horizontal translation)

d: vertical shift $d = -1$ down 1 unit

$$y = 2\cos\frac{\pi}{2}x - 3$$

cos x

Amp = 2

period : $\frac{2\pi}{\frac{\pi}{2}} = 4$

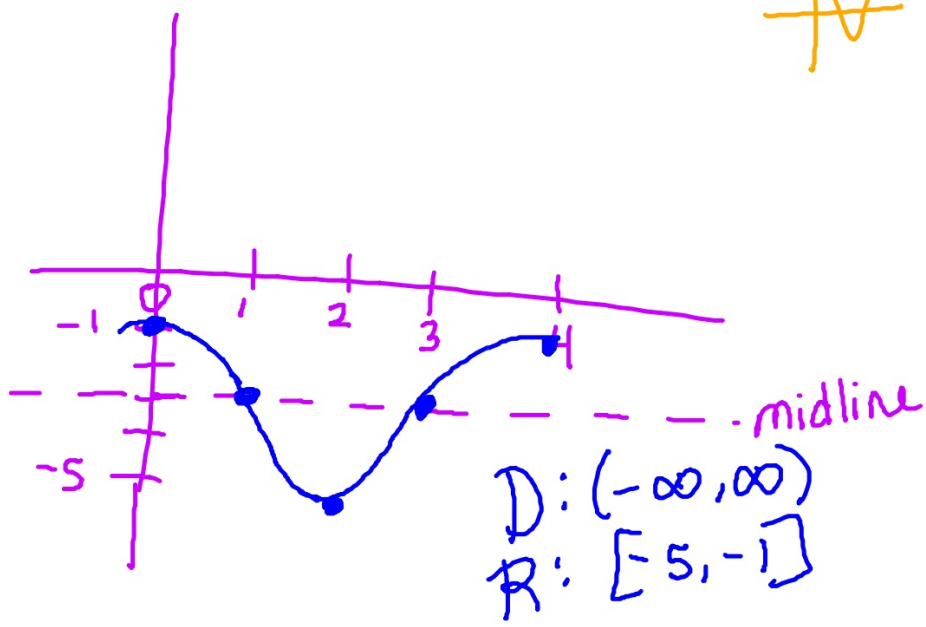
$b = \frac{\pi}{2}$

Vert. ↓ 3

increments

$\frac{\text{period}}{4}$

$\frac{4}{4} = 1$



D: $(-\infty, \infty)$

R: $[-5, -1]$

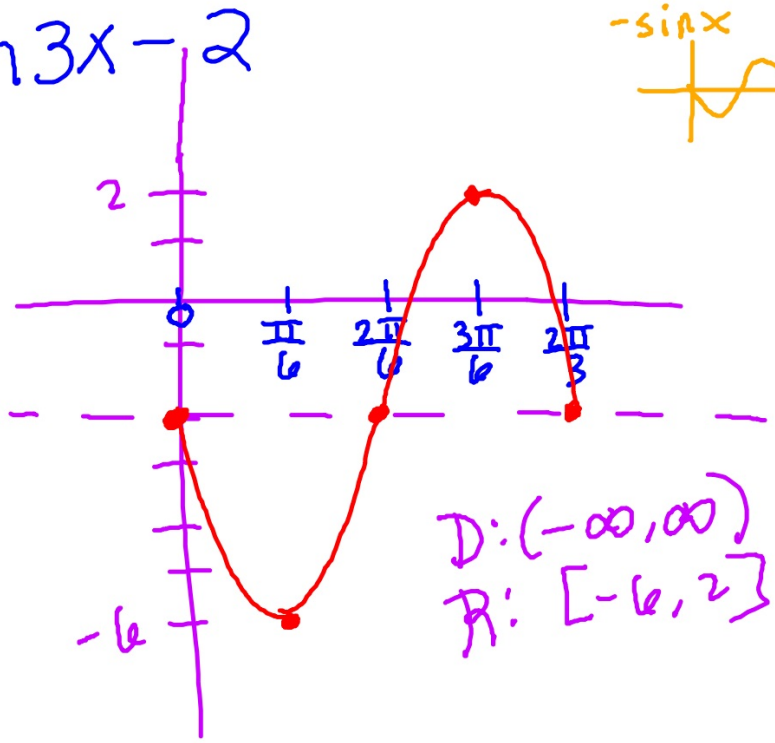
$$y = -4\sin 3x - 2$$

Amp : 4

period : $\frac{2\pi}{3}$

vert. ↓ 2

increments : $\frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$



D: $(-\infty, \infty)$

R: $[-6, 2]$

How to find the phase shift

$$y = a\sin(bx - c) + d$$

or

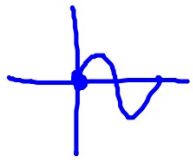
$$y = a\sin b(x - c) + d$$

Horizontal translation

Phase shift (PS)

$bx - c = 0$	$b(x + c) = 0$
$y = 2\cos(2x - \pi) - 1$	$y = 3\sin 2(x + 8)$
$2x - \pi = 0$	$2(x + 8) = 0$
$x = \frac{\pi}{2}$	$x = -8$
$\rightarrow \frac{\pi}{2}$	$\leftarrow 8$

$$y = \sin\left(x + \frac{\pi}{2}\right) + 1$$



Amp: 1

period: $\frac{2\pi}{1} = 2\pi$

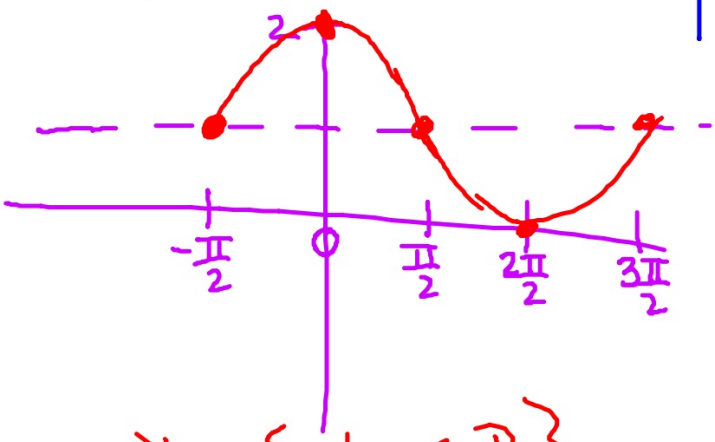
vert. $\uparrow$ 1

PS (horiz)

$x = -\frac{\pi}{2}$

$\leftarrow \frac{\pi}{2}$

increments: $\frac{\pi}{2}$



D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid 0 \leq y \leq 2\}$

$$y = -2\cos(4x - \pi) - 3$$

D: $(-\infty, \infty)$

R: $[-5, -1]$

Amp: 2

period: $\frac{\pi}{2}$

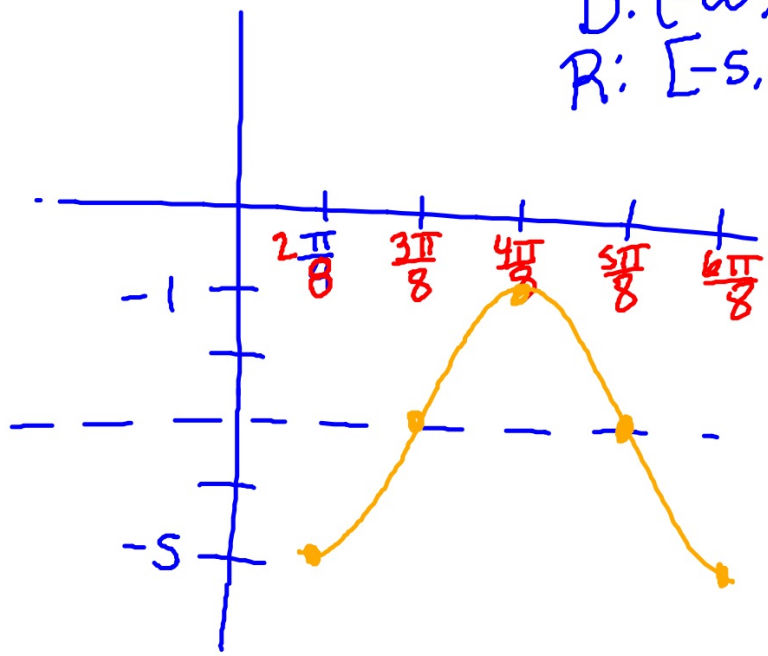
vert. ↓ 3

PS: horiz → $\frac{\pi}{4}$

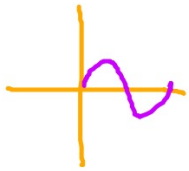
$4x - \pi = 0$

$x = \frac{\pi}{4}$ Start here

increments: $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$



$$y = 2\sin(4x+\pi) - 3$$



Amp. = 2

period: $\frac{\pi}{2}$

$b = 4$

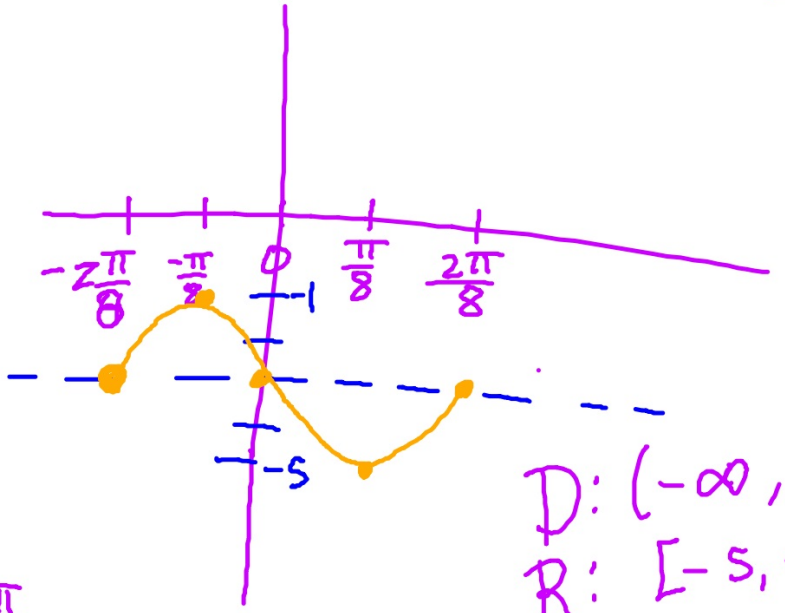
vert: ↓ 3

horiz: ← $\frac{\pi}{4}$

$4x + \pi = 0$

$x = \frac{-\pi}{4}$

increments: $\frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$



D: $(-\infty, \infty)$

R: $[-5, -1]$

$$y = 3\sin 2x - 1$$

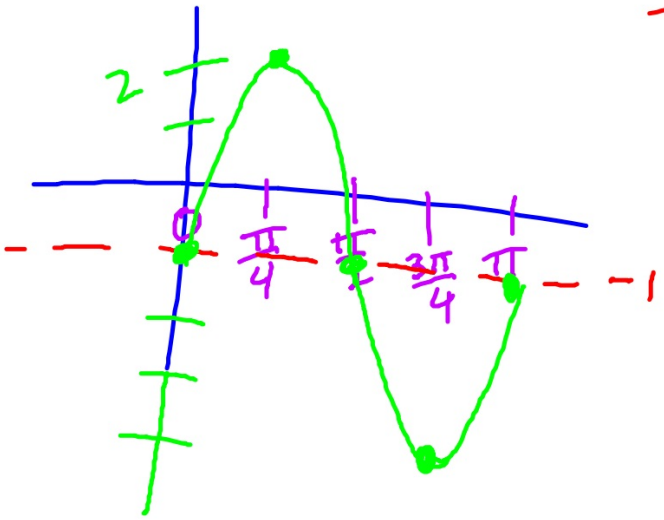
Amp: 3

period: π

$\downarrow$ 1

increments

$$\frac{\text{period}}{4} = \frac{\pi}{4}$$



$y = \sin x$

$y = \cos x$

$y = -\sin x$

$y = -\cos x$

② $y = -4\sin 3x - 2$

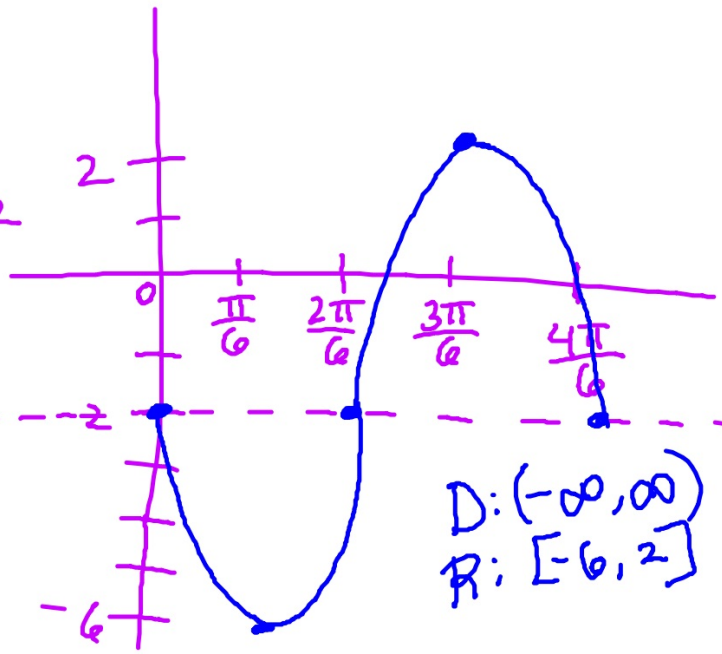
Amp: 4

period: $\frac{2\pi}{3}$

Vert. shift ↓ 2

<u>Increments</u>

$\frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$



midline (vertical shift)

D: $(-\infty, \infty)$

R: $[-6, 2]$