1.8 Quadratic Formula Cont.
Quadratic Words Problems

<u>Recall the Quadratic Formula:</u>

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex 1: Solve.

a) $x^2 + 3x + 1 = 3$

b) $-x^2 + 5x - 4 = x + 1$

$-x^2 + 4x - 5 = 0$

$-1(x^2 - 4x + 5) = 0$

$a = 1$
$b = -4$
$c = 5$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} = 2 \pm i$$

$$\frac{4 \pm 2i}{2} = \frac{4}{2} \pm \frac{2i}{2}$$

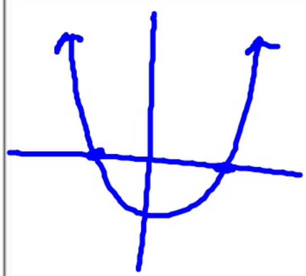
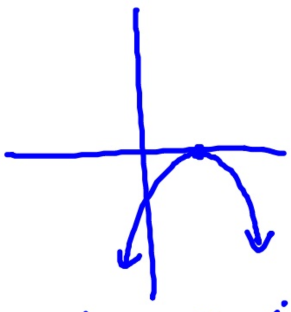
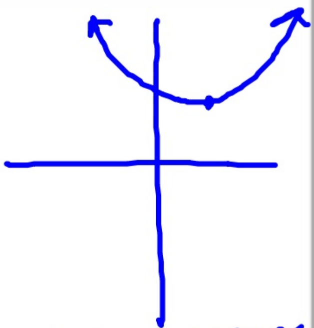
c) $x^2 = 6x - 4$

$x^2 - 6x + 4 = 0$

The Discriminant:

- In the quadratic formula, the expression b^2-4ac is called the discriminant.
- The discriminant is used to determine the types of solutions for the quadratic equation.

Using The Discriminant: $D = b^2 - 4ac$

Value of discriminant	$D > 0$	$D = 0$	$D < 0$
Number of solutions	2	1	2
Type of solutions	real	real	imaginary
Graph of $y = ax^2 + bx + c$		 vertex on x-axis	 never crosses x-axis

ex 2: Find the discriminant and give the number and type of solutions of the equation.

a) $x^2 - 8x + 13 = -4$

$x^2 - 8x + 17 = 0$

$D = b^2 - 4ac$

$= 8^2 - 4(1)(17)$

$= 64 - 68$

$= -4$

2 imaginary solutions

b) $x^2 - 8x + 16 = 0$

$D = 64 - 4(1)(16)$

$= 0$

1 real solution

c) $8x^2 - 2x + 1 = x^2 + 6$

$7x^2 - 2x - 5 = 0$

$D = (-2)^2 - 4(7)(-5)$

$= 4 + 140$

$= 144$

2 real

Fun fact: if the discriminant is zero or a perfect square, the equation is FACTORABLE.

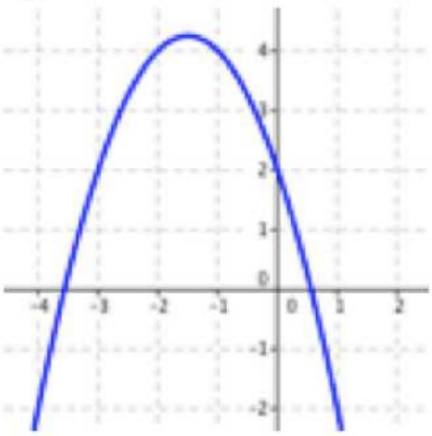
$7x^2 - 2x - 5 = 0$

$(7x + 5)(x - 1) = 0$

$x = -\frac{5}{7}, 1$

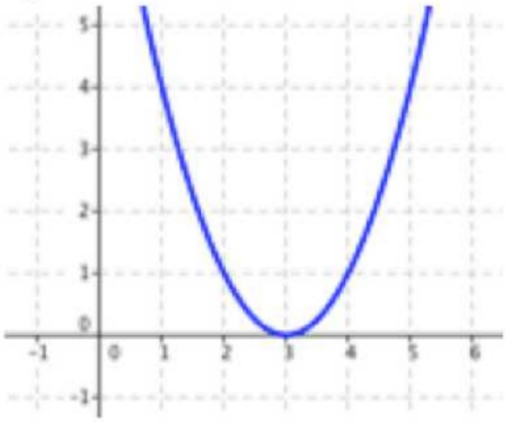
ex 3: The graph of $y = ax^2 + bx + c$ or the solutions of $ax^2 + bx + c = 0$ are given. Determine if the discriminant is positive, negative, or zero. Explain your reasoning.

a)



positive;
parabola intercepts the X-axis at 2 points.

b)



0; parabola's vertex is on the x-axis.

c) $x = 2 \pm 3i$

Negative; Imaginary solutions mean the discriminant is less than 0.

ex 4: Consider the quadratic equation: $3x^2+12x+c=0$

Find all values of c for which the equation has...

a) two real solutions (when $D>0$)

$$b^2-4ac>0$$
$$144-4(3)(c)>0$$
$$144-12c>0$$
$$-12c>-144$$
$$\boxed{c<12}$$

$a=3$
$b=12$
$c=c$

b) one real solution $D = 0$

$$b^2 - 4ac = 0$$

$$12^2 - 4(3)(c) = 0$$

$$144 - 12c = 0$$

$$c = 12$$

c) two imaginary solutions

$D < 0$

$b^2 - 4ac < 0$

$12^2 - 4(3)c < 0$

$144 - 12c < 0$

$-12c < -144$

$c > 12$

$144 < 12c$

$12 < c$

ex 5: Determine which method is best to solve each quadratic equation. Do not repeat a method. DO NOT SOLVE.

a) $2x^2 - 6x + 1 = 0$

b) $-2x^2 - 7x + 15 = 0$

c) $6 - 2(x+1)^2 = 0$

d) $3x^2 - 12x - 14 = 0$

Factor
Sq. root
Quad form
CTS

ex 5: Determine which method is best to solve each quadratic equation. Do not repeat a method. ~~DO NOT SOLVE.~~

a) $2x^2 - 6x + 1 = 0$ $\quad X = \frac{6 \pm 2\sqrt{7}}{4} = \frac{3}{2} \pm \frac{\sqrt{7}}{2}$

b) $-2x^2 - 7x + 15 = 0$ $\quad -(2x^2 + 7x - 15) = 0$

$X = \frac{3}{2}, -5$ $\quad -(2x - 3)(x + 5) = 0$

c) $6 - 2(x+1)^2 = 0$ $\quad 3 = (x+1)^2$

$-1 \pm \sqrt{3}$ $\quad \pm\sqrt{3} = x + 1$

d) $3x^2 - 12x - 14 = 0$ $\quad 3(x^2 - 4x \underline{+4}) - 14 \underline{-12} = 0$

$3(x-2)^2 = 26$

$X = 2 \pm \frac{\sqrt{78}}{3}$

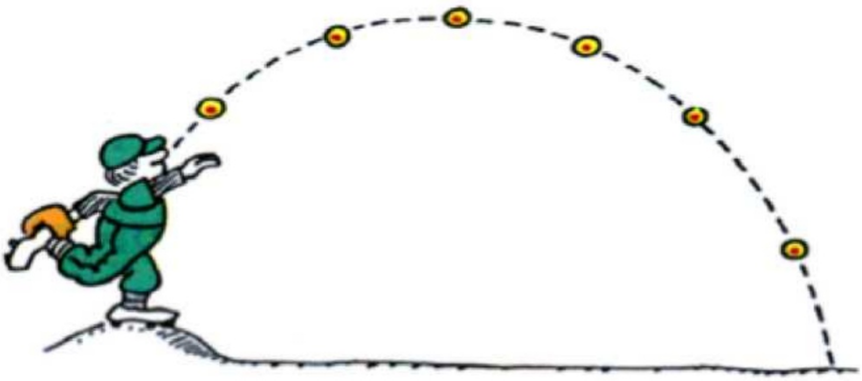
Quadratic Word Problems

Review:

- When given a quadratic function in standard form, how do you find the vertex? $x = \frac{-b}{2a}$; plug in this valve to get y

- How do you find the maximum/minimum value of quadratic function? y-valve of vertex

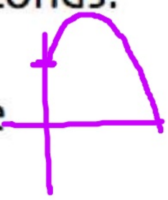
Falling Objects



1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$

where $\underbrace{h(t) \text{ is the height}}_{y}$ in feet and $\underbrace{t}_{\text{"x"}}$ is the time in seconds.

a) What is the initial height of the rocket? What is the height of the rocket after 1 sec?

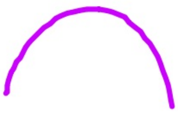


$t = 0$; $h(0) = 160$ ft

$t = 1$; $h(1) = -16 + 48 + 160 = 192$ ft

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$
where h(t) is the height in feet and t is the time in seconds.

b) At what time does the rocket reach its maximum height?



vertex

$$x = \frac{-b}{2a} = \frac{-48}{2(-16)} = \frac{3}{2} = 1.5 \text{ sec}$$

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$
where h(t) is the height in feet and t is the time in seconds.

c) What is the maximum height?

$$h\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 160$$

$$= -\frac{\overset{4}{\cancel{16}} \cdot 9}{\cancel{4}\,1} + \frac{\overset{24}{\cancel{48}} \cdot 3}{\cancel{2}\,1} + 160$$

$$= -36 + 72 + 160 = 196 \text{ ft}$$

1. The height of a rocket launched upward from a 160-foot cliff is modeled by $h(t) = -16t^2 + 48t + 160$
where h(t) is the height in feet and t is the time in seconds.

d) At what time does the rocket hit the ground?

ground
0ft

$0 = -16t^2 + 48t + 160$

$0 = -16(t^2 - 3t - 10)$

$-16(t-5)(\cancel{t+2})$

$t = 5$ sec

2. The height of a flare fired from the deck of a ship in distress can be modeled by $h = -16t^2 + 112t + 56$ water in feet and t is the time in seconds. At what time(s) will the flare be at a height of 56 feet?

$$\cancel{56} = -16t^2 + 112t + \cancel{56}$$

$$0 = -16t(t - 7)$$

$$t = 0, 7 \text{ sec}$$

$$\begin{array}{r} \overset{4}{16} \\ \times\ 7 \\ \hline 112 \end{array}$$