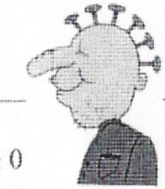
Algebra 2 – Things to Remember!



Exponents:

$x^0 = 1$

$x^{-m} = \dfrac{1}{x^m}$

$x^m \cdot x^n = x^{m+n}$

$(x^n)^m = x^{n \cdot m}$

$\dfrac{x^m}{x^n} = x^{m-n}$

$\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$(xy)^n = x^n \cdot y^n$

Complex Numbers:

$\sqrt{-1} = i$ $\sqrt{-a} = i\sqrt{a}; a \ge 0$

$i^2 = -1$ $i^{14} = i^2 = -1$ divide exponent by 4, use remainder, solve.

$(a+bi)$ conjugate $(a-bi)$

$(a+bi)(a-bi) = a^2 + b^2$

Standard Form Of A Complex Number: $a+bi$

a = "real part" b = "imaginary part"

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Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ natural log

$e = 2.71828\ldots$

$\log x = \log_{10} x$ common log

Change of base formula:

$\log_b a = \dfrac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\log_b 1 = 0$

$\log_b(m \cdot n) = \log_b m + \log_b n$

$\log_b\left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^r) = r\log_b m$

Domain: $\log_b x$ is $x > 0$

Factoring:

Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b+c)$

$x^2 - a^2 = (x-a)(x+a)$

$(x+a)^2 = x^2 + 2ax + a^2$

$(x-a)^2 = x^2 - 2ax + a^2$

Factor by Grouping:

$x^3 + 2x^2 - 3x - 6$

$(x^3 + 2x^2) - (3x+6)$ group

$x^2(x+2) - 3(x+2)$ factor each

$(x^2 - 3)(x+2)$ factor

Exponentials $e^x = \exp(x)$

$b^x = b^y \rightarrow x = y$ $(b > 0$ and $b \ne 1)$

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x$ $e^{\ln x} = x$

$\ln e = 1$ $e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.)

Solve by factoring, completing the square, quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac > 0$ two real unequal roots

$b^2 - 4ac = 0$ repeated real roots

$b^2 - 4ac < 0$ two complex roots

↑discriminant↑

Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$

Completing the square: $3x^2 - 6x - 1 = 0$

1. Rewrite the equation by factoring out the leading coefficient, a, from the quadratic and linear terms.	$3(x^2 - 2x) - 1 = 0$
2. Rewrite with place holders.	$3(x^2 - 2x + \_\_\_\_) + \_\_\_\_ - 1 = 0$
3. CTS:	$\left(\dfrac{b}{2}\right)^2 = \left(\dfrac{-2}{2}\right)^2$
4. Add the CTS value inside and subtract it outside. (*Consider the L.C. when filling in the 2nd placeholder.)	$3(x^2 - 2x + 1) - 3 - 1 = 0$
5. Factor and simplify.	$3(x-1)^2 - 4 = 0$
6. Solve using square roots.	$x = 1 \pm \dfrac{2\sqrt{3}}{3}$

Absolute Value: $|a| > 0$

$|a| = \begin{cases} a; & a \ge 0 \\ -a; & a < 0 \end{cases}$

$|m| = b \Rightarrow m = -b$ or $m = b$

$|m| < b \Rightarrow -b < m < b$

$|m| > b \Rightarrow m > b$ or $m < -b$

Sum and Difference of Squares:

$a^2 + b^2 = PRIME$

$a^2 - b^2 = (a+b)(a-b)$

Sum and Difference of Cubes:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

"SOAP"

Perfect Square Trinomials:

$a^2 + 2ab + b^2 = (a+b)^2$

ex: $x^2 - 6x + 9 = (x-3)^2$

ex: $4x^2 + 20x + 25 = (2x+5)^2$

Radicals: Remember to use fractional exponents.

$\sqrt[a]{x} = x^{\frac{1}{a}}$ $\quad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

$\sqrt[n]{a^n} = a$ $\quad \sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: look for perfect powers.

$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$

$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \cdot 9x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2}$

Use conjugates to rationalize denominators:

$\frac{5}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$2x - 5\sqrt{x} - 3 = 0 \quad \rightarrow \quad (2x-3)^2 = (5\sqrt{x})^2$

$4x^2 - 12x + 9 = 25x \quad \rightarrow \textit{solve}: x = 9;\ x = 1/4$

CHECK ANSWERS. Answer only $x = 9$.

Functions: A function is a set of ordered pairs in which each x-element has only ONE y-element associated with it.

Vertical Line Test: is this graph a function?
Domain: x-values used; **Range:** y-values used
Onto: all elements in B used.
1-to-1: no element in B used more than once.
Composition: $(f \circ g)(x) = f(g(x))$
Inverse functions f & g: $f(g(x)) = g(f(x)) = x$
Horizontal line test: will inverse be a function?

Transformations:
$-f(x)$ over x-axis; $f(-x)$ over y-axis
$f(x+a)$ horizontal shift; $f(x)+a$ vertical shift
$f(ax)$ stretch horizontal; $af(x)$ stretch vertical

Working with Rationals (Fractions):
Simplify:
remember to look for a factoring of -1:

$\frac{3x-1}{1-3x} = \frac{-1(-3x+1)}{1-3x} = -1$

Add: Get the common denominator.
Factor first if possible:
Multiply and Divide: Factor First

Common Pythagorean Triplets:
3, 4, 5
5, 12, 13
8, 15, 17
7, 24, 25

Sequences
Arithmetic: $a_n = a_1 + (n-1)d$

$S_n = \frac{n(a_1 + a_n)}{2}$

Geometric: $a_n = a_1 \cdot r^{n-1}$

$S_n = \frac{a_1(1-r^n)}{1-r}$

Recursive: Example:
$a_1 = 4;\quad a_n = 2a_{n-1}$

Recursive Rule for Arithmetic Sequences
$a_1 = \#$ $\quad a_n = a_{n-1} + d$

Recursive Rule for Arithmetic Sequences
$a_1 = \#$ $\quad a_n = r \cdot a_{n-1}$

Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.

$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$

multiply all by $2x^2 - 9x - 5$ *and get*

$22 - 3(x-5) = 2(2x+1)$

$22 - 3x + 15 = 4x + 2$

$37 - 3x = 4x + 2$

$35 = 7x$

$5 = x$

Great! But the only problem is that $x = 5$ does not CHECK!!!! There is no solution. Extraneous root.
Motto: Always CHECK ANSWERS.

Equations of Circles: $x^2 + y^2 = r^2$ center origin
$(x-h)^2 + (y-k)^2 = r^2$ center at (h,k)
$x^2 + y^2 + Cx + Dy + E = 0$ general form

Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom

$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{2-4x}{x^2} \cdot \frac{x^2}{4x-2} = -1$

Method 2: Mult. all terms by common denominator for all.

$\frac{\frac{2}{x^2} - \frac{4}{x}}{\frac{4}{x} - \frac{2}{x^2}} = \frac{x^2 \cdot \frac{2}{x^2} - x^2 \cdot \frac{4}{x}}{x^2 \cdot \frac{4}{x} - x^2 \cdot \frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$

Quadratic Functions:

Standard Form: $f(x) = ax^2 + bx + c$

Vertex In Standard Form:

$$x = -\frac{b}{2a} \qquad y = f\left(-\frac{b}{2a}\right)$$

Vertex Form: $f(x) = a(x-h)^2 + k$

Vertex In Vertex Form: (h,k)

Methods for Finding Zeros of Quadratic Functions:

- Factoring
- Square roots
- CTS
- Quadratic Formula

Polynomial Functions:

Methods for Finding Zeros of Polynomials Functions:

- Factoring
- Rational Root Theorem (P/Q's)

End Behavior:
**Consider the Degree and Leading Coefficient*

	INFORMAL:	FORMAL:
• Even Degree • Negative L.C.	↓↓	$x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to -\infty$
• Even Degree • Positive L.C.	↑↑	$x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to \infty$
• Odd Degree • Negative L.C.	↑↓	$x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to -\infty$
• Odd Degree • Positive L.C.	↓↑	$x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$

Synthetic Substitution: $f(x) = 2x^3 - x + 5$

$f(3) = ?$

3	2	0	-1	5	*Therefore,* $f(3) = 56$
	↓	6	18	51	
	2	6	17	56	

Radians and Degrees

Change to radians multiply by $\frac{\pi}{180}$

Change to degrees multiply by $\frac{180}{\pi}$

Quadrantal Angles:

$0^\circ, 90^\circ, 180^\circ, 270^\circ$

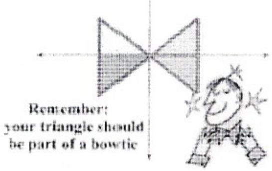
$0,\ \pi/2,\ \pi,\ 3\pi/2,\ 2\pi$

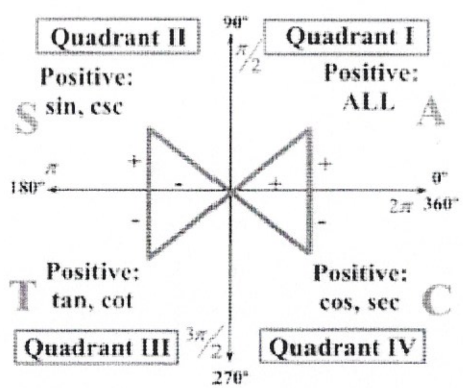
Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

Even/Odd Identities:

$\sin(-\theta) = -\sin\theta$	*Odd*
$\cos(-\theta) = \cos\theta$	*Even*
$\tan(-\theta) = -\tan\theta$	*odd*

Reference triangles are drawn to the x-axis.





Trig Functions

$\sin\theta = \frac{o}{h};\ \cos\theta = \frac{a}{h};\ \tan\theta = \frac{o}{a}$

$\csc\theta = \frac{h}{o};\ \sec\theta = \frac{h}{a};\ \cot\theta = \frac{a}{o}$

Reciprocal Functions

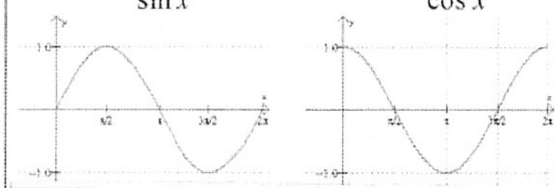
$\sin\theta = \frac{1}{\csc\theta};\ \cos\theta = \frac{1}{\sec\theta};\ \tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta};\ \sec\theta = \frac{1}{\cos\theta};\ \cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Trig Graphs

$\sin x$ $\qquad$ $\cos x$



7 steps to Sketch Sine and Cosine

$y = a\sin(bx - c) + d$

$y = a\cos(bx - c) + d$

1. **amplitude:** $|a|$
2. **period:** $P = \frac{2\pi}{|b|}$
3. **increment:** $I = \frac{P}{4}$
4. **phase shift**(Hor. Shift): Set $bx - c = 0$ and solve for x.
5. **midline:** $y = d$
6. **Pattern:**
   - **Sine: AMAmA**
   - **Cosine: MAmAM**
7. **5 Key Coordinates:** Start with the Phase Shift then add the increment 4 times

Inverses

To find an Inverse:

Ex: $f(x) = x^3 + 4$

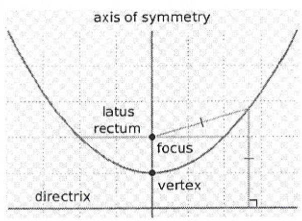
1. switch x and y	$x = y^3 + 4$
2. solve for y	$y = \sqrt[3]{x-4}$
3. label appropriately	$f^{-1}(x) = \sqrt[3]{x-4}$

Graphs of $f(x)$ and $f^{-1}(x)$ Properties:

- The graph of $f(x)$ and $f^{-1}(x)$ are reflections about the line $y = x$
- The domain of $f(x)$ is the range of $f^{-1}(x)$
- The range of $f(x)$ is the domain of $f^{-1}(x)$
- If $f(x)$ contains the point (a, b) then $f^{-1}(x)$ contains the point (b, a)

Conic Sections

Parabolas



Standard Form

$(x-h)^2 = 4p(y-k)$ If p>0, opens upward If p<0, opens downward FUNCTION	$(y-k)^2 = 4p(x-h)$ If p>0, opens right If p<0, opens left NOT A FUNCTION
Vertex: (h, k) $\lvert p \rvert$ = distance from the vertex to the focus/directrix Latus rectum: $\lvert 4p \rvert$	

Circles

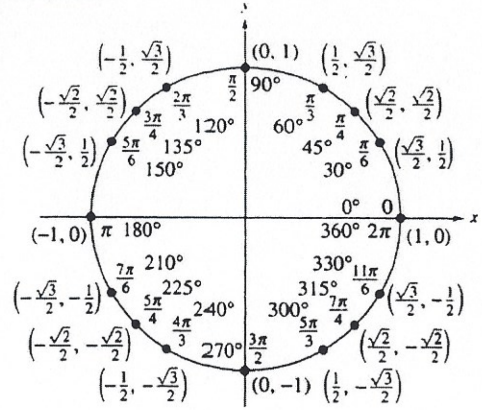
Standard Form: $(x-h)^2 + (y-k)^2 = r^2$

Center: (h, k)   Radius: r

Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Unit Circle



Domain Restrictions

Fractions:

$y = \dfrac{f(x)}{g(x)}$	Domain: $\{x \mid g(x) \neq 0\}$

Ex: $y = \dfrac{x^2 + 9}{x - 2}$, $\{x \mid x \neq 2\}$

Ex: $y = \dfrac{x - 2}{x^2 + 9}$, $\{x \mid x \in R\}$

Even Roots:

$y = \sqrt[n]{f(x)}$, n is even	Domain: $\{x \mid f(x) \geq 0\}$

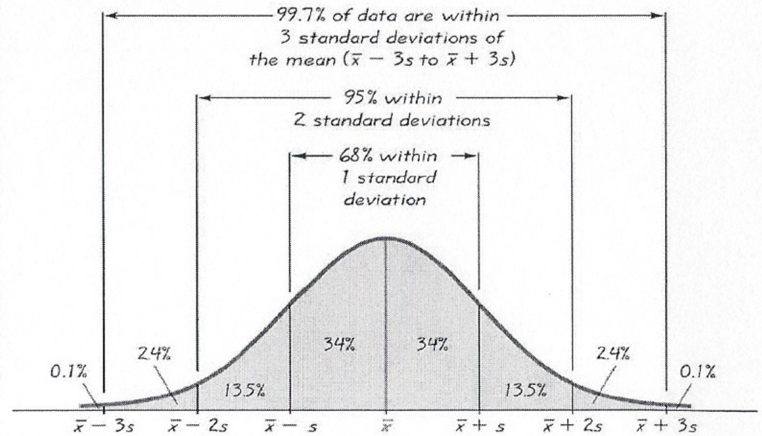
Ex: $y = 2\sqrt{x+7} - 3$, $\{x \mid x \geq -7\}$

Logarithms:

$y = \log_b(f(x))$	Domain: $\{x \mid f(x) > 0\}$

Ex: $y = -2\ln x + 4$, $\{x \mid x > 0\}$

Empirical Rule



Experimental Probability: $P(E) = \dfrac{\#\text{ of times event } E \text{ occurs}}{\text{total } \#\text{ of observed outcomes}}$

Theoretical Probability: $P(E) = \dfrac{\#\text{ of outcomes in } E}{\text{total } \#\text{ of outcome in the sample space}}$

$P(A \text{ and } B) = P(A) \cdot P(B)$ *For independent events*

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ *For dependent events*

$P(A \text{ or } B) = P(A) + P(B)$ *For mutually exclusive events*

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ *For inclusive events*

Set Theory

Set: A collection of elements

$A = \{1,2,3\} \quad B = \{0,3\} \quad C = \{0,1,2,3,4\}$

Intersection: ($\cap$) objects that belong to set A *AND* B.

$A \cap B = \{3\}$

Union: ($\cup$) objects that belong to set A *OR* B.

$A \cup B = \{0,1,2,3\}$

Complement: ($\sim A = A' = \bar{A}$) objects that *DO NOT* belong to set A but do belong to set C.

A' within set $C = \{0,4\}$

Null or Empty Set: ($\{\ \}$ or $\varnothing$) the set that contains no elements.