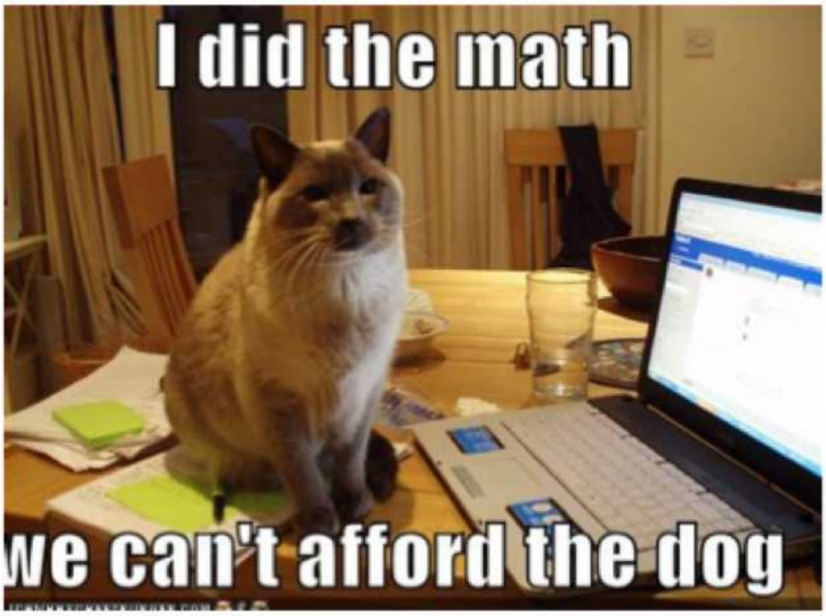
4.4 Evaluating Logarithms
4.5 Properties of Logarithms



ex: Evaluate.

a) 2^5 32

b) $81^{3/4}$ 27

c) $9^{-5/2}$ $\frac{1}{243}$

d) $-16^{5/4}$ -32

ex: Solve.

a) $2^x = 16$ 4

b) $3^x = \frac{1}{3}$ -1

c) $71^x = 1$ 0

d) $25^x = 5$ 1/2

e) $27^x = 9$ 2/3

Definition of a Logarithm

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

ex: Rewrite in exponential form.

a) $\log_3 9 = 2$ exponent

$3^2 = 9$

b) $\log_{22} 1 = 0$

$22^0 = 1$

ex: Rewrite in logarithmic form.

a) $3^5 = 243$

$\log_3 243 = \underline{5}$

$2^x = 8$

$2^x = 12$

b) $27^{-2/3} = \frac{1}{9}$

$\log_{27} \frac{1}{9} = -\frac{2}{3}$

ex: Evaluate.

a) $\log_4 64 = 3$

b) $\log_3 81 = 4$

c) $\log_5 25 = 2$

d) $\log_7\left(\frac{1}{7}\right) = -1$

e) $\log_{13} 1 = 0$

$\log_4 64 = \square$

$4^{\square} = 64$

$3^{\square} = 81$

ex: Evaluate.

f) $\log_{25} 5$

$\frac{1}{2}$

g) $\log_5\left(\frac{1}{125}\right) = -3$

$5^x = \frac{1}{125}$

h) $\log_{81} 27 = 3/4$ $\quad 81^x = 27$

i) $\log_2(-4)$

$2^{\square} = -4$ not poss.

j) $\log_{25}\left(\frac{1}{5}\right) = -\frac{1}{2}$

$\log_4 4^{20} = 20$

k) $\log_{\star}\left(\star^{100}\right) = 100$

$\star > 0, \star \neq 1$

Special Logarithms

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm	**Natural Logarithm**
$\log_{10} x = \log x$	$\log_e x = \ln x$

Most calculators have keys for evaluating common and natural logarithms.

$e \approx 2.718$

$\log_e x = \ln x$

$\boxed{\cancel{\ln_e x}}$

ex: Evaluate.

a) $\log 100 = 2$

b) $\log\left(\frac{1}{10}\right) = -1$

c) $\log .001 = \log \frac{1}{1000} = -3$

ex: Evaluate.

d) $\ln 1 = 0$ $\qquad$ $\log_5 1$

$e^0 = 1$

e) $\ln\left(\frac{1}{e}\right) = -1$ $\qquad$ $\ln e^{-1} = -1$

f) $\ln e^2 = 2$ $\qquad$ $\log_2 2 = 1$

g) $\ln e = 1$

 ex: Evaluate on your calculator.

a) $\log 16 = 1.204$

$10^{1.204} \approx 16$

$\log 10 = 1$

$\log 100 = 2$

b) $\ln 7 = 1.9459$

Logarithms and Exponentials are INVERSES!

$f(x) = \log_b x \qquad g(x) = b^x$

ex: Evaluate.

a) $(f \circ g)(x) = \log_b b^x = x$

b) $(g \circ f)(x) \quad b^{\log_b x} = x$

$2^{\log_2 16}$

2^4

16

ex: Evaluate.

a) $7^{\log_7 x}$ $= x$

b) $\log_{62} 62^x$ $= x$

c) $\log 10^x$ $= x$

ex: Evaluate.

d) $e^{\ln 7} = 7$

e) $\log_5 25^x = x\log_5 25$

$\log_5 5^{2x} = 2x$

f) $\log_{64} 4^y = y(\log_{64} 4) = \frac{1}{3}y$

$\frac{y}{3}$

$\log a^b = b\log a$

REVIEW - Exponent Properties

$$b^m \cdot b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^n = b^{mn}$$

Logarithm Properties

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Logarithm properties are used to EXPAND and CONDENSE logarithmic expressions.

$$\log xy = \log x + \log y \quad \checkmark$$

$$\log(x+y) =$$

$$\log(x^2-16) = \log(x+4)(x-4) = \log(x+4) + \log(x-4)$$

ex: Expand.

a) $\log_3\left(\frac{abc}{9d}\right)$

$\log_3 a + \log_3 b + \log_3 c - (\log_3 9 + \log_3 d)$

$\log_3 a + \log_3 b + \log_3 c - 2 - \log_3 d$

ex: Expand.

b) $\log_5\left(\frac{a^2b^3}{c^4}\right)$

$\log_5 a^2 + \log_5 b^3 - \log_5 c^4$

$2\log_5 a + 3\log_5 b - 4\log_5 c$

$\boxed{\ln x = \log_e x}$

$\cancel{\ln_e x}$

c) $\log\left(\frac{100a^2}{b^3c}\right)$

$2 + 2\log a - 3\log b - \log c$

ex: Expand.

d) $\ln\left(\frac{1}{ab^2c^3}\right) = \ln 1 - \ln a - \ln b^2 - \ln c^3$

$= 0 - \ln a - 2\ln b - 3\ln c$

e) $\log_3\left(a+b^2\right)$ ☺

ex: Expand.

f) $\log_4\left(\dfrac{a+b}{a-b^2}\right)$ $= \log_4(a+b) - \log_4(a-b^2)$

g) $\log_2\left(a^2-b^2\right)$ $= \log_2(a+b)(a-b)$

$\log_2(a+b) + \log_2(a-b)$

ex: Expand.

h) $\log_3 (a-b)^7 = 7\log_3(a-b)$

i) $\ln \sqrt{\dfrac{y^3+z}{x^3(a+1)^5}} = \dfrac{1}{2}\ln\left(\dfrac{y^3+z}{x^3(a+1)^5}\right)$

$= \dfrac{1}{2}\left[\ln(y^3+z) - 3\ln x - 5\ln(a+1)\right]$

j.) $\log_2 \sqrt[3]{\dfrac{16a^5}{b^2+c^2}}$

$$\log_2\left(\frac{16a^5}{b^2+c^2}\right)^{1/3}$$

$$\frac{1}{3}\left[\log_2 16 + \log_2 a^5 - \log_2(b^2+c^2)\right]$$

$$\frac{1}{3}\left[4 + 5\log_2 a - \log_2(b^2+c^2)\right]$$

k.) $\log_{32}\left(\frac{x^3-y^3}{8}\right)$

$3/5$

$\log_{32}(x^3-y^3)-\log_{32}8$

$\log_{32}(x-y)+\log_{32}(x^2+xy+y^2)-3/5$

ex: Condense.

a) $2\log_5 a - 3\log_5 b + 4\log_5(c+d)$

$$\log_5 a^2 - \log_5 b^3 + \log_5 (c+d)^4$$

$$\log_5\left(\frac{a^2(c+d)^4}{b^3}\right)$$

ex: Condense.

b) $\frac{1}{2}\log x+\frac{3}{2}\log y-10\log z$

$$\log x^{1/2}+\log y^{3/2}-\log z^{10}$$

$$\log\left(\frac{x^{1/2}y^{3/2}}{z^{10}}\right)=\log\left(\frac{\sqrt{xy^{3}}}{z^{10}}\right)$$

ex: Condense.

c) $-3\log x - 4\log y - \frac{2}{3}\log z$

$$\log\left(\frac{1}{x^3 y^4 z^{2/3}}\right)$$