5.1, 5.3, 6.1 Extra Practice

1.

In which of the following models is $\dfrac{dy}{dt}$ directly proportional to y?

I. $y = e^{kt} + C$

II. $y = Ce^{kt}$

III. $y = 28^{kt}$
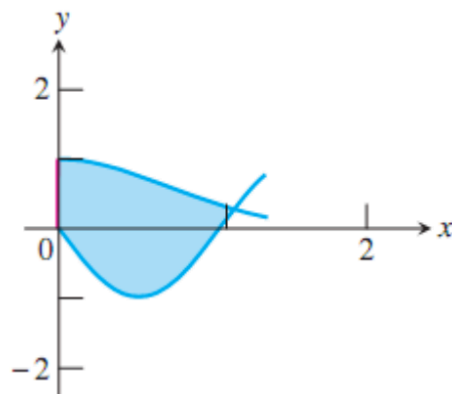
IV. $y = 3\left(\dfrac{1}{2}\right)^{3t+1}$

(A) I only   (B) II only   (C) I and II only   (D) II and III only   (E) II, III, and IV   (F) all of them

2.

(Calculator permitted) Let R be the shaded region enclosed by the graphs of $y = e^{-x^2}$, $y = -\sin(3x)$, and the y-axis as shown at right.
Which of the following gives the approximate area of the region R?

(A) 1.139   (B) 1.445   (C) 1.869   (D) 2.114   (E) 2.340

3.

If $\dfrac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \dfrac{1}{2}$?

(A) $-\dfrac{1}{2}\ln 2$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{2}\ln 2$ (D) $\dfrac{\sqrt{2}}{2}$ (E) $\ln 2$

4.

(**Calculator permitted**) Population y grows according to the equation $\dfrac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069   (B) 0.200   (C) 0.301   (D) 3.322   (E) 5.000

5.

Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \dfrac{1}{x}$. Which of the following gives the area of the region enclosed by the graphs of f and g between $x = 1$ and $x = 2$?

(A) $e^2 - e - \ln 2$ (B) $\ln 2 - e^2 + e$ (C) $e^2 - \dfrac{1}{2}$ (D) $e^2 - e - \dfrac{1}{2}$ (E) $\dfrac{1}{e} - \ln 2$

6.

Let R be the region in the first quadrant bounded by the x-axis, the graph of $x = y^2 + 2$, and the line $x = 4$. Which of the following integrals gives the area of R?
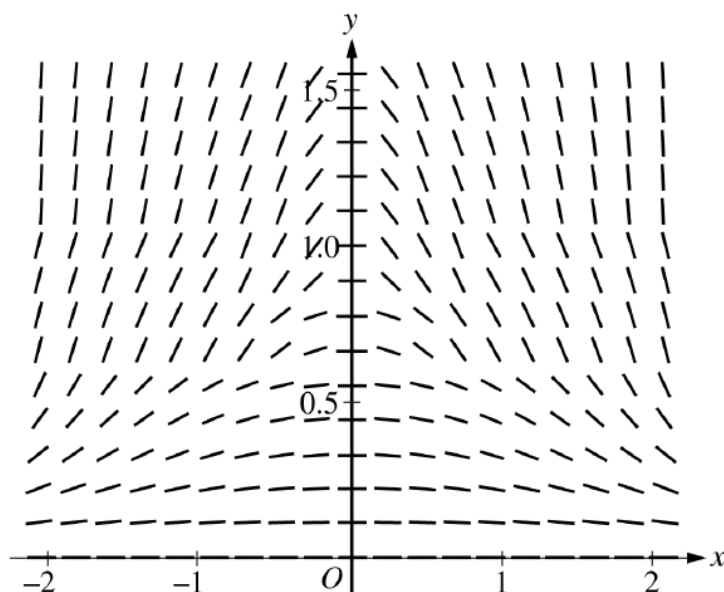
(A) $\displaystyle\int_0^{\sqrt{2}} \left[4 - \left(y^2 + 2 \right) \right] dy$ 　　(B) $\displaystyle\int_0^{\sqrt{2}} \left[\left(y^2 + 2 \right) - 4 \right] dy$ 　　(C) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - \left(y^2 + 2 \right) \right] dy$

(D) $\displaystyle\int_{-\sqrt{2}}^{\sqrt{2}} \left[\left(y^2 + 2 \right) - 4 \right] dy$ 　　(E) $\displaystyle\int_2^4 \left[4 - \left(y^2 + 2 \right) \right] dy$

7.

Which of the following gives the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$.

(A) 2 　　(B) $\dfrac{9}{2}$ 　　(C) $\dfrac{13}{2}$ 　　(D) 13 　　(E) $\dfrac{27}{2}$

8.

The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

9.

Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

10.

If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

11.

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\dfrac{e}{2} - 1$ (D) $\dfrac{e-1}{2}$ (E) $e - 1$