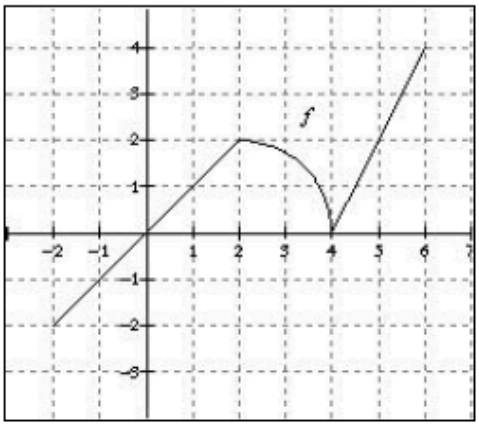
AP Calculus AB - 2nd Fundamental Theorem/Accumulation Functions

1. Let $F(x) = \int_2^x f(t)\,dt$. The graph of f on the interval $[-2, 6]$ consists of two line segments and a quarter of a circle, as shown at right.



(a) Find $F(0)$ and $F(4)$.

(b) Determine the interval where $F(x)$ is increasing. Justify your answer.

(c) Find the critical numbers of $F(x)$ and determine if each corresponds to a relative minimum value, a relative maximum value, or neither. Justify your answers.

(d) Find the absolute extreme values of $F(x)$ and the x-values at which they occur. Justify your answers.

(e) Find the x-coordinates of the inflection points of $F(x)$. Justify your answer.

(f) Determine the intervals where the graph of $F(x)$ is concave down. Justify your answer.

2) Evaluate each expression.

a. $\frac{d}{dx}\left[\int_1^x \sqrt{t^2-1}\,dt\right]$

b. $\frac{d}{dx}\left[\int_x^3 t\sin t\,dt\right]$

c. $\frac{d}{dx}\left[\int_{\pi/2}^{x^2} \cos t\,dt\right]$

3)

Use the Second Fundamental Theorem of Calculus to find the derivatives of the following functions.

a) $f(x)=\int_{1}^{x}\left(t^2+1\right)^{20}\,dt$

b) $g(x)=\int_{-1}^{x}\sqrt{t^3+1}\,dt$

c) $g(x)=\int_{\pi}^{x}\frac{1}{1+t^4}\,dt$

d) $f(x)=\int_{4}^{x^2}\cos\left(t^2\right)\,dt$

4)

Find the interval on which the curve $y=\int_{0}^{x}\left(t^3+t^2+1\right)\,dt$ is concave up. Justify your answer.

Answers

1. (a) $F(0) = \int_2^0 f(t)\,dt = -\int_0^2 f(t)\,dt = -2$

$F(4) = \int_2^4 f(t)\,dt = \pi$

(b) $F(x)$ is increasing when $F'(x)$ is positive.
Since $F'(x) = f(x)$ and $f(x)$ is positive on the intervals (0,4) and (4,6), then $F(x)$ is increasing on these intervals and in fact on the interval [0,6].

(c) At $x = 0$, $F(x)$ has a relative minimum because $F' = f$ changes from negative to positive there.
At $x = 4$, $F(x)$ has neither a relative minimum nor a relative maximum because $F' = f$ does not change sign there.

(d) Using the Candidates Test, we compare $F(-2) = 0$, $F(0) = -2$, $F(4) = \pi$, and $F(6) = \pi + 4$, and find that $F(0) = -2$ is the absolute minimum value of F on the interval $[-2, 6]$ and $F(6) = \pi + 4$ is the absolute maximum value of F on the interval $[-2, 6]$.

(e) $F(x)$ has inflection points at $x = 2$ and $x = 4$ because $F'' = f'$ changes signs at these points.

(f) The graph of $F(x)$ is concave down on the interval (2, 4) because $F'(x) = f(x)$ is strictly decreasing on this interval and $F''(x) = f'(x)$ is defined there, or, equivalently, because $F''(x)$ is negative on the interval.

2)

a. $\sqrt{x^2 - 1}$ b. $-x \sin x$ c. $2x \cos x^2$

3)

a. $f'(x) = (x^2 + 1)^{20}$ b. $g'(x) = \sqrt{x^3 + 1}$ c. $g'(x) = \dfrac{1}{1 + x^4}$

d. $f'(x) = 2x \cos x^4$

4) The function is concave up on the intervals $(-\infty, -2/3) \cup (0, \infty)$ because y″ > 0 on these intervals.