## 2.5 Implicit Differentiation

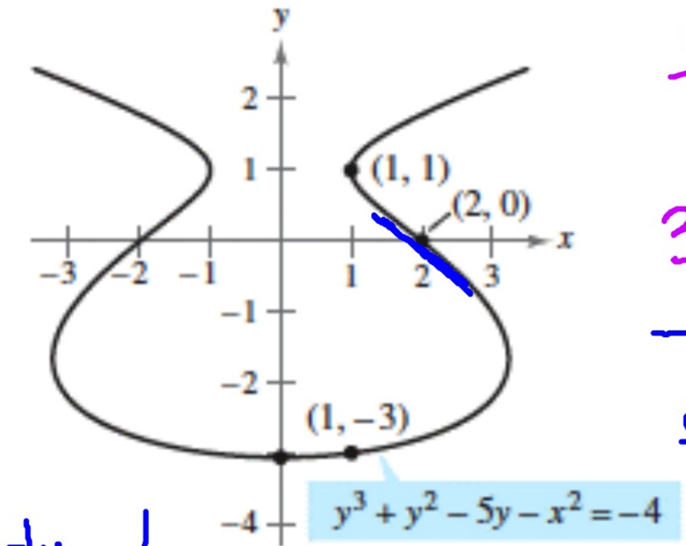
- Distinguish between functions written in implicit form and explicit form.
- Use implicit differentiation to find the derivative of a function.

| *Implicit Form* | *Explicit Form* | *Derivative* |
|---|---|---|
| $xy = 1$ | $y = \frac{1}{x} = x^{-1}$ | $\frac{dy}{dx} = -x^{-2} = -\frac{1}{x^2}$ |

Up to now, we hae been finding derivatives of functions explicitly. You can also find derivatives of equations that are not functions implicitly. Or, if you wish, you can find derivatives of functions implicitly too :)

$$x^2y - y^3 + x^5 = 7$$

Using implicit differentiation, we can find the derivative of equations like:




$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{4}{-5}$$

And, we can find the slope at a given point.

$$\frac{d}{dx}\left(y^3 + y^2 - 5y - x^2 = -4\right)$$

$$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x\cancel{\frac{dx}{dx}}$$

$$\frac{dy}{dx}\left(3y^2 + 2y - 5\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$$

Find the derivative implicitly.

#2 $\frac{d}{dx}\left(x^2 - y^2 = 25\right)$

$2x\cancel{\frac{dx}{dx}} - 2y\frac{dy}{dx} = 0$

$x - y\frac{dy}{dx} = 0$

$-y\frac{dy}{dx} = -x$

$\frac{dy}{dx} = \frac{x}{y}$

Find the equation of the tangent line
at the given point.

#3 $\frac{d}{dx}\left(x^3 + y^3 = 6(xy) - 1,\right)$ (2, 3)

$xy$

$y'$

$$3x^2\cancel{\frac{dx}{dx}} + 3y^2\frac{dy}{dx} = 6\left(x\cdot 1\cdot\frac{dy}{dx} + y\cdot 1\cdot\cancel{\frac{dx}{dx}}\right)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 - 6x\right) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{2y - x^2}{y^2 - 2x}$$

$$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{6-4}{9-4} = \frac{2}{5}$$

$$\boxed{y - 3 = \frac{2}{3}(x - 2)}$$

Find the slope at the given point.

#4 $\frac{d}{dx}\left(x \cos y = 1\right)$, $\left(2, \frac{\pi}{3}\right)$

$\cos(y)$

$$x \cdot \left(-\sin y \cdot \frac{dy}{dx}\right) + \cos y \cdot 1 \cancel{\frac{dx}{dx}} = 0$$

$$-x \sin y \frac{dy}{dx} = -\cos y$$

$$\frac{dy}{dx} = \frac{\cos y}{x \sin y} = \frac{\cot y}{x}$$

$$\left.\frac{dy}{dx}\right|_{\left(2, \frac{\pi}{3}\right)} = \frac{\cot \frac{\pi}{3}}{2} = \frac{\frac{\sqrt{3}}{3}}{2} = \frac{\sqrt{3}}{6}$$

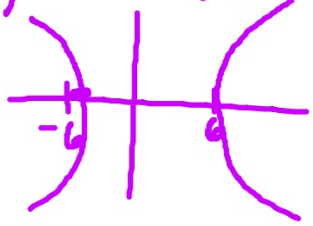
**In Exercises 17–20, (a) find two explicit functions by solving the equation for *y* in terms of *x*, (b) sketch the graph of the equation and label the parts given by the corresponding explicit functions, (c) differentiate the explicit functions, and (d) find *dy*/*dx* and show that the result is equivalent to that of part (c).**

#5 $x^2 - y^2 = 36$

$y = \pm\sqrt{x^2-36}$

$y = \pm\frac{1}{2}(x^2-36)^{-1/2}\cdot 2x$

$y' = \pm\frac{x}{\sqrt{x^2-36}}$ explicit




$\frac{d}{dx}(x^2 - y^2 = 36)$ implicit

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y} = \frac{x}{\pm\sqrt{x^2-36}}$

#6: Given $x^2 + y^2 = 4$, show that $\frac{d^2y}{dx^2} = \frac{-4}{y^3}$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1)\frac{dx}{dx} - (-x)\cdot 1\frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x\frac{dy}{dx}}{y^2} = \frac{\left(-y + x\left(\frac{-x}{y}\right)\right)\cdot y}{y^2 \cdot y}$$

$$= \frac{-y^2 - x^2}{y^3} = \frac{-(y^2+x^2)}{y^3} = \frac{-4}{y^3}$$

9.) $$\frac{d}{dx}\left(x^3 - 3(x^2y) + 2(xy^2) = 12\right)$$

$$3x^2 - 3\left(x^2\frac{dy}{dx} + y\cdot 2x\right) + 2\left(x\cdot 2y\frac{dy}{dx} + y^2\right) = 0$$

$$3x^2 - 3x^2\frac{dy}{dx} - 6xy + 4xy\frac{dy}{dx} + 2y^2 = 0$$

$$\frac{dy}{dx}\left(-3x^2 + 4xy\right) = 6xy - 3x^2 - 2y^2$$

$$\frac{dy}{dx} = \frac{6xy - 3x^2 - 2y^2}{-3x^2 + 4xy}$$

11. $$\frac{d}{dx}\left(\sin x + 2\cos(2y) = 1\right)$$

$$\cos x - 2\sin(2y)\cdot \underset{=}{2}\cdot\frac{dy}{dx} = 0$$

$$\cos x - 4\sin(2y)\frac{dy}{dx} = 0$$

$$\frac{\cos x}{4\sin(2y)} = \frac{dy}{dx}$$