2.4 The Chain Rule

- Find the derivative of a composite function using the Chain Rule.
- Find the derivative of a function using the General Power Rule.
- Simplify the derivative of a function using algebra.
- Find the derivative of a trigonometric function using the Chain Rule.

1. $f(x) = x^2(2x-3)^4$

$$f'(x) = x^2 \cdot 4(2x-3)^3 \cdot 2 + (2x-3)^4 \cdot 2x$$

$$= \underline{8x^2(2x-3)^3} + \underline{2x(2x-3)^4}$$

$$= 2x(2x-3)^3(4x + 2x-3)$$

$$= 2x(2x-3)^3(6x-3) = 6x(2x-3)^3(2x-1)$$

#4 $y = \frac{x}{\sqrt{x^4 + 4}} = x(x^4 + 4)^{-1/2}$

$$y' = x\left(-\frac{1}{2}(x^4+4)^{-3/2} \cdot 4x^3\right) + (x^4+4)^{-1/2} \cdot 1$$

$$= -2x^4(x^4+4)^{-3/2} + (x^4+4)^{-1/2}$$

#2

$$g(\theta) = \cos^2 8\theta = (\cos 8\theta)^2$$

$$g'(\theta) = 2(\cos 8\theta)^1 \cdot (-\sin 8\theta) \cdot 8$$

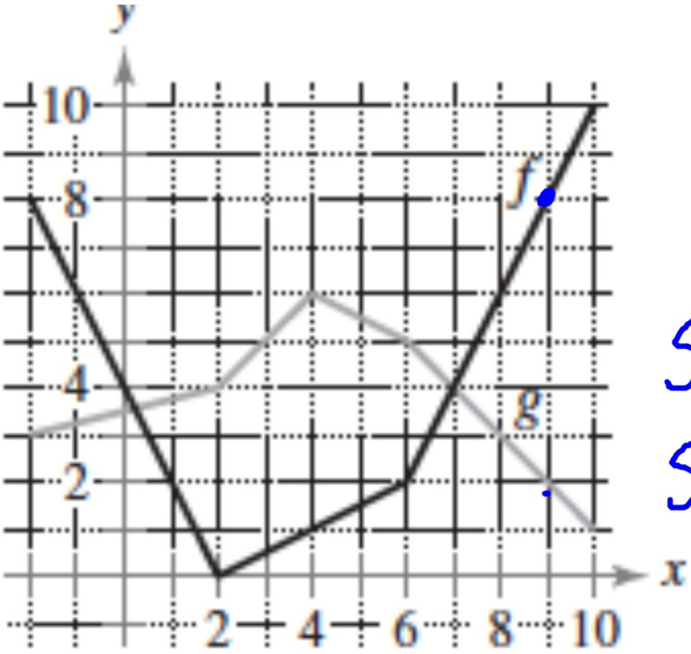
$$= -16 \cos 8\theta \sin 8\theta$$

$$g(x) = \sin^4 3x = (\sin 3x)^4$$

$$g'(x) = 4(\sin 3x)^3 \cos 3x \cdot 3$$

$$= 12 \sin^3 3x \cos 3x$$

#3 **In Exercises 109 and 110, the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.**



Find $s'(9)$.

$s(x) = g(f(x))$

$s'(x) = g'(f(x)) \cdot f'(x)$

$s'(9) = g'(f(9)) \cdot f'(9)$

$g'(8) \cdot 2$

$(-1) \cdot 2$

$s'(9) = -2$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

#4 $h(x) = 3(g(x))^3$; Find h'(2)

#5 **t(x)=f(x)/g(x); Find t'(2)**

#6 **p(x) = f(g(x)); Find p'(2)**

47.) $g(x) = 5(\tan 3x)$

$g'(x) = 5\sec^2 3x \cdot 3$

$y = c \cdot f(x)$

$y' = c f'(x)$

55.) $y = 4(\sec x)^2$

$y' = 8(\sec x)^1 \cdot \sec x \tan x$

9.) $h(x) = \dfrac{\sqrt{x}}{x^3+1} = x^{1/2}(x^3+1)^{-1}$

$h'(x) = x^{1/2} \cdot \left(-1(x^3+1)^{-2} \cdot 3x^2\right) + (x^3+1)^{-1} \cdot \frac{1}{2} x^{-1/2}$

$= \frac{-6}{2} x^{5/2} (x^3+1)^{-2} + \frac{1}{2} x^{-1/2} (x^3+1)^{-1}$

$= \frac{-1}{2} x^{-1/2} (x^3+1)^{-2} \left(6x^3 - (x^3+1)^1\right)$

$= \dfrac{-(5x^3-1)}{2\sqrt{x}(x^3+1)^2}$

75.) $f(x) = \dfrac{x^2}{x-1}$

$(0, 0)$

$(2, 4)$

$$f'(x) = \frac{(x-1)2x - x^2 \cdot 1}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$$

$$0 = \frac{x(x-2)}{(x-1)^2}$$

47.) $y = \frac{3}{2}\left(\frac{1-\sin x}{\cos x}\right) = \frac{3}{2}\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)$

$$y' = \frac{3}{2}\left(\frac{\cos x(-\cos x) - (1-\sin x)(-\sin x)}{\cos^2 x}\right) \qquad \frac{-3}{2(1+\sin x)}$$

$$= \frac{3}{2}\left(\frac{-\cos^2 x + \sin x - \sin^2 x}{\cos^2 x}\right)$$

$$\frac{3}{2}\left(\frac{\sin x - 1}{1-\sin^2 x}\right) = \frac{3}{2}\left(\frac{\cancel{\sin x - 1}}{(1+\sin x)\cancel{(1-\sin x)}}\right)^{-1}$$