2.2 Basic Differentiation Rules and Rates of Change

- Find the derivative of a function using the Constant Rule.
- Find the derivative of a function using the Power Rule.
- Find the derivative of a function using the Constant Multiple Rule.
- Find the derivative of a function using the Sum and Difference Rules.
- Find the derivatives of the sine function and of the cosine function.
- Use derivatives to find rates of change.

Find the slope

Find the equation of a tangent line

Find where a function has a horizontal slope

Find constants so that a function is differentiable everywhere

#1 Find the slope at the indicated point.

$f(x) = x^2 + 3x + 4, \quad (-2, 2)$

$x \quad y$

$f'(x) = 2x + 3$

$f'(-2) = -1$

$f(x) = \dfrac{4}{\sqrt{x}} \quad (4,2)$

$f(x) = 4x^{-1/2}$

$f'(x) = -2x^{-3/2}$

$f'(4) = -2 \cdot 4^{-3/2}$

$= -2 \cdot (2^2)^{-3/2}$

$= -2 \cdot 2^{-3} = -2 \cdot \frac{1}{8}$

$= \frac{-1}{4}$.

#2 Find the equation of the tangent line at the point indicated.

$f(x) = x^2 + 3x + 4, \quad \underline{(-2, 2)}$

$f'(-2) = \underline{-1}$

point-slope form

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = -1(x+2)}$

numerical

$\cancel{y - 2 = (2x+3)(x+2)}$

$f(x) = \dfrac{4}{\sqrt{x}} \quad (4,2)$

$f(4) = -\frac{1}{4}$

$\boxed{y - 2 = -\frac{1}{4}(x-4)}$

#3 Find the equation of the line tangent to the graph of f and parallel to the given line.

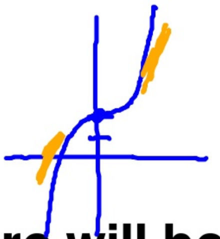
$f(x) = x^3 + 2$ $\qquad$ $3x - y - 4 = 0$

$f'(x) = 3x^2$

$3 = 3x^2$

$1 = x^2$

$\pm 1 = x$



$y = 3x - 4$

$y' = 3$

There will be two points that have a slope of 3.

$(1, 3)$ $m = 3$

$\boxed{y - 3 = 3(x - 1)}$

$(-1, 1)$ $m = 3$

$\boxed{y - 1 = 3(x + 1)}$

#4

Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

$y = x^3 - 6x^2 + 1$

zero slope

where the function has slope 0.

$y' = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

$(0, 1)$ $(4, -31)$

#5 Find a and b so that f(x) is differentiable.

$$f(x)=\begin{cases} ax^2-3x & x<1 \\ bx-2 & x\geq 1 \end{cases}$$

differentiability implies continuity

Continuity

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\lim_{x\to 1^-}(ax^2-3x) = \lim_{x\to 1^+}(bx-2) = f(1)$

$a-3 = b-2$

Differentiability

$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x) = f'(1)$

$\lim_{x\to 1^-}(2ax-3) = \lim_{x\to 1^+}(b) = f'(x$

$2a-3 = b$

$a-3 = (2a-3)-2$

$a-3 = 2a-5$

$2 = a$

$b = 2a-3$

$b = 2(2)-3$

$b = 1$